Renewable and Sustainable Energy Reviews 25 (2013) 670–686

Contents lists available at SciVerse ScienceDirect

Renewable and Sustainable Energy Reviews

journal homepage: www.elsevier.com/locate/rser

Empirical and theoretical correlations on viscosity of nanofluids: A review

L. Syam Sundar [a,*], K.V. Sharma [c], M.T. Naik [c], Manoj K. Singh [a,b]

[a] Center for Mechanical Technology and Automation, Department of Mechanical Engineering, University of Aveiro, 3810-193 Aveiro, Portugal
[b] Aveiro Institute of Nanotechnology, University of Aveiro, 3810-193 Aveiro, Portugal
[c] Department of Mechanical Engineering, J N T U College of Engineering, Hyderabad, India

ARTICLE INFO

Article history:
Received 25 January 2013
Received in revised form
12 April 2013
Accepted 20 April 2013
Available online 11 June 2013

Keywords:
Correlations
Nanofluid
Particle size
Temperature
Volume concentration

ABSTRACT

In the past decade nanotechnology has developed in many directions. Nanofluid is a mixture of nanosized particles dispersed in fluids. Nanofluids are new generation heat transfer fluids used in heat exchangers for energy conservation. Viscosity is an important property particularly concerning fluids flowing in a tube in heat exchangers. In this regard, an attempt has been made to review the available empirical and theoretical correlations for the estimation of viscosity of nanofluids. The review also extended to preparation of nanofluids, nanoparticle volume concentration, nanofluid temperature, particle size and type of base fluid on viscosity of nanofluids. The available experimental results clearly indicate that with the dispersion of nanoparticles in the base fluid viscosity increases and it further increases with the increase in particle volume concentration. Viscosity of nanofluid decreases with increase of temperature.

Contents

1. Introduction

Conventional heat transfer fluids such as water, engine oil, transformer oil, ethylene glycol and propylene glycol play an important role in many industries such as power generation, chemical production, air-conditioning, transportation, microelectronics etc. Several heat transfer enhancement techniques are used to improve the heat transfer rate of such fluids. Those techniques are change of flow geometry and boundary conditions; improving thermophysical properties of fluids like increase of thermal conductivity is used. Thermal conductivity of solids is higher than fluids. Because of high thermal conductivity of solids, many studies

* Corresponding author.
E-mail address: sslingala@rediffmail.com (L.S. Sundar).

1364-0321/$ - see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.rser.2013.04.003

have been conducted on the thermal behavior of solid particles dispersed in fluids. The existing classical models available in the literature for the estimation of thermal conductivity of solid–fluid mixtures are Maxwell's [1] and Hamilton–Crosser model [2], but there is no experimental evidence to prove these models.

Dispersion of ultrafine magnetic solid particles in the fluids was first introduced by Akoh et al. [3] who also estimated their magnetic properties. Later, Ahuja [4] measured the thermal conductivity and viscosity of 50 μm and 100 μm polystyrene spheres dispersed in aqueous sodium chloride and glycerin and obtained 3 times thermal conductivity enhancement compared to base fluid. Both the above mentioned researches considered microsize particles and also observed particle agglomeration in the base fluids. In a similar way researchers like Choi and Tran [5] and Choi et al. [6,7] at Argonne National Laboratory, USA, have developed advanced fluids for industrial applications, including district heating and cooling systems and they also found particle agglomeration in the base fluid. Masuda et al. [8] also used ultrafine particles of Al_2O_3, SiO_2 and TiO_2 dispersed in fluids for the estimation of thermal conductivity and viscosity and found better enhancement. Even though, they found better enhancement with dispersion of particles, they also observed particles agglomeration in the base fluid. The problem of particle agglomeration is solved by Choi [9] and his team by inventing nanometer sized solid particles in fluids called as 'nanofluid'. For the preparation of nanofluids, commonly used nanoparticles are metals (Al, Ag, Cu, Ni etc.), metal-oxides (Al_2O_3, CuO, Fe_2O_3, Fe_3O_4, SiO_2, TiO_2 etc.), some other compounds (Al_2Cu, Ag_2Cu, Ag_2Cu, Ag_2Al, AlN, SiC, graphene, carbon nanotubes etc.) and commonly used base fluids are water, ethylene glycol, propylene glycol, transformer oil, engine oil etc.

Many experimental investigations are available for thermal conductivity enhancement of nanofluids. Lee et al. [10] considered Al_2O_3 and CuO nanofluids and found better thermal conductivity enhancement compared to base fluid. Choi et al. [11] observed 160% thermal conductivity enhancement with CNTs dispersed in synthetic poly(α-olefin) oil at 1.0% volume concentration.

The thermal properties of nanofluids like thermal conductivity, viscosity, density and specific heat are very important, before introducing the nanofluids in devices like heat exchangers and condensers. Based on the solid–fluid homogeneous models, the properties like density and specific heat can be estimated. The other properties like thermal conductivity and viscosity can be estimated experimentally. For a nanofluid flowing in a tube or any equipment viscosity of the fluid plays an important role, because Reynolds number of the fluid depends on viscosity. Viscosity explains the internal resistance between the fluid layers. In the laminar flow or turbulent flow, the pressure drop of the fluid is directly proportional to the viscosity of fluid and it also influences the convective heat transfer coefficient. So, viscosity is also a very important property like thermal conductivity, whenever a system is involved in a fluid flow [12].

In recent years, lot of research progressed on the nanofluids related to thermal conductivity [13–22], forced convective heat transfer in a tube [23–32], forced convective heat transfer in a tube with inserts [33–41], natural heat transfer [42–44], mixed convection [45], boiling heat transfer [46–50], heat exchangers [51–55], solar flat plate collectors [56–58], car radiators [59], slip mechanism [60], electrical conductivity [61], cooling of electronic devices [62]. Some review papers [63–66] have emphasized on the thermal conductivity of nanofluids. Very few review papers are available on the viscosity of nanofluids [67].

During experimental investigations on viscosity of nanofluids, Pak and Cho [68] investigated the viscosity of TiO_2/water and Al_2O_3/water nanofluids and observed 3% and 200% enhancement respectively compared to base fluid. Masuda et al. [69] measured the viscosity of TiO_2 27 nm nanofluid and found 60% enhancement at 4.3% volume concentration compared to water. Bobbo et al. [70] investigated the viscosity of SWCNT/water and TiO_2/water nanofluids and found 12.9% and 6.8% enhancement at 1.0% volume concentration at 283 K respectively. Lee et al. [71] estimated the viscosity of Al_2O_3/water nanofluid and observed 2.9% enhancement at 0.3% volume concentration at a temperature of 21 °C. Wang et al. [72] estimated the viscosity of Al_2O_3 and CuO nanoparticles dispersed in water, vacuum pump fluid, engine oil and ethylene glycol and found 30% enhancement with Al_2O_3/water nanofluid at 3% volume concentration. They also reported that the enhancement of viscosity is similar in Al_2O_3/water nanofluid and Al_2O_3/ethylene glycol nanofluid. Chadwick et al. [73] studied the rheological behavior of titanium dioxide (uncoated anatase) in ethylene glycol and found viscosity enhancement with increase of particle volume concentration. Kwak and Kim [74] observed thermal conductivity and viscosity enhancement with CuO nanoparticles dispersed in ethylene glycol. Teipel and Forter-Barth [75] prepared paraffin oil and hydroxyl hydroxyterminated polybutadiene (HTPB) oil based aluminum nanofluid and observed that paraffin oil/aluminum suspensions exhibit non-Newtonian flow behavior over a wide range of concentrations, whereas the HTPB/aluminum suspensions exhibit Newtonian behavior up to 50% volume concentration. Katiyar et al. [76] prepared paraffin oil based Fe–Ni nanofluid and measured the viscosity in the 10% weight concentration. Prasher et al. [77] observed the viscosity enhancement for Al_2O_3/propylene glycol nanofluid; Chen et al. [78] found the viscosity enhancement for TiO_2/water, TiO_2/ethylene glycol nanofluid, TNT/water nanofluid and TNT/ethylene glycol nanofluids; Murshed et al. [79] observed the viscosity enhancement for ethylene glycol based TiO_2 and Al_2O_3 nanofluids.

The application of water based nanofluids is limited in subzero countries like Alaska, Canada, Northern Europe and Russia, because water can freeze at 0 °C. This can be overcome by adding small ratio of ethylene gylcol or propylene gylcol to water. By adding these fluids to water the freezing point of water can be reached to −35 °C. Kulkarni et al. [80] first time measured the convective heat transfer and viscosity of 60:40% ethylene glycol and water mixture based CuO, Al_2O_3 and SiO_2 nanofluids and also evaluated the performance of these nanofluids in the heating buildings in cold regions. Naik and Sundar [81] also prepared CuO nanofluids by considering 70:30% propylene gylcol/water mixture as a base fluid for the estimation of thermal conductivity and viscosity enhancement. Sundar et al. [82] prepared 50:50% ethylene glycol/water mixture based Al_2O_3 and CuO nanofluids for the estimation of thermophysical properties.

In the available literature most of the researchers have explained the viscosity of nanofluids with the effect of volume concentration and temperature. Some review papers discussed the viscosity of nanofluid but they mostly concentrated on the thermal conductivity of nanofluids. Review papers like Das et al. [83] have given the importance of nanofluid viscosity, Keblinski et al. [84], Daungthongsuk and Wongwises [85] mentioned the viscosity of nanofluid for convective heat transfer, Sridhar and Satapathy [86] have given the importance to viscosity of Al_2O_3 based nanofluids. The mentioned reviews are not sufficient for completely understanding the viscosity behavior of nanofluids. In this regard a complete study is required to cover all the aspects of viscosity of nanofluids.

In this regard, the present review paper focuses on viscosity of different kinds of nanofluids, with effects of base fluids, volume concentration, temperature, particle size, theoretical models and developed correlations.

Absolute viscosities of different commonly used fluids are summarized in Table 1. The available literature on viscosity of nanofluids with various parameters is shown in Table 2.

Table 1
Viscosity of some common liquids at a temperature of 30 °C.

Fluid	Absolute viscosity, (Pa s)
Acetic acid	0.001155
Acetone	0.000316
Alcohol, propyl	0.00192
Benzene	0.000601
Bromine	0.00095
Carbon disulfide	0.00036
Carbon tetrachloride	0.00091
Castor oil	0.650
Chloroform	0.00053
Decane	0.000859
Dodecane	0.00134
Ethanol	0.001095
Ether	0.000223
Ethylene glycol	0.0162
Freon refrigerant R-11	0.00042
Glycerin	0.950
Heptane	0.000376
Hexane	0.000297
Kerosene	0.00164
Linseed oil	0.0331
Methanol	0.00056
Mercury	0.0015
Octane	0.00051
Phenol	0.0080
Propane	0.00011
Propylene	0.00009
Propylene glycol	0.042
Toluene	0.000550
Turpentine	0.001375
Water, Fresh	0.00089
20:80% EG/W	0.0013
40:60% EG/W	0.00226
60:40% EG/W	0.00384
30:70% PG/W	0.00219

2. Methods for preparation of nanofluids

2.1. Single-step method

Preparation of nanofluid is very important before estimating the viscosity of nanofluids, because particle sedimentation in the base fluid purely depends on the preparation method. Nanofluids are not a simple mixture of solid particles dispersed in liquids. Some special preparation method is required to obtain uniform suspension, stable suspension, and less sedimentation. Nanofluids are produced by dispersing metal, metal-oxides, non-metals of nanometer size particles in the base fluids like water, ethylene glycol, propylene glycol, engine oil etc.

The single-step method involves condensing nanophase powders from the vapour phase directly into a flowing low vapour pressure liquid i.e. nanoparticles are made and dispersed in liquid simultaneously. The nanoparticles are prepared either by using physical vapour deposition method or liquid chemical method. The single step method of nanofluids was first prepared by Akoh et al. [3] by direct evaporation approach method, which is called the vacuum evaporation onto a running oil substrate. The idea of their method is to develop nanoparticles, but the difficulty is to separate the nanoparticles from the fluids to produce dry nanoparticles. Wagener et al. [87] modified the vacuum evaporation onto a running oil substrate method by employing high pressure magnetron sputtering for the preparation of suspensions with metal nanoparticles such as silver (Ag) and iron (Fe). Eastman et al. [13] also developed a modified vacuum evaporation onto a running oil substrate method, in which Cu nanoparticles are directly condensed into nanoparticles by flowing low vapor pressure liquid like ethylene glycol. Zhu et al. [88] developed a new single step method for the preparation of copper nanofluids by chemical reduction of $CuSO_4 \cdot 5H_2O$ with $NaH_2PO_2 \cdot H_2O$ in ethylene glycol under microwave irradiation. Zhu et al. [89] prepared stable CuO nanofluids by using wet chemical method with a particle diameter range 15–50 nm and also studied various parameters like copper salts reaction time. Lo et al. [90] developed submerged arc nanoparticle synthesis system for the preparation of copper nanofluids by considering that pure copper is heated and vaporized by arc sparking between two electrodes which are immersed in dielectric liquids such as water, pure ethylene glycol, 30%, 50%, 70% volume of ethylene glycol mixed with de-ionized water and they produced needle-like structure with an average width of 20 nm and length of 80 nm. Lo et al. [91] also produced Ni nanomagnetic fluid by using submerged arc nanoparticle synthesis system by dispersing Ni nanoparticles in water, ethylene glycol and 50% of water and ethylene glycol mixture with an average particle diameter of 30, 20 and 10 nm respectively. The advantage for this method is that the particle sedimentation is very less and the disadvantage is that only low vapor pressure fluids are compatible.

2.2. Two-step method

In the two-step method, nanoparticles are separated from the dispersed fluid. Basically chemical co-precipitation method is used to synthesize the nanoparticles. This method involves a reaction between reagent metal slats reduced in the dispersant. Generally used dispersants are water, ethylene glycol, glycerin, acetone, methanol, ethanol etc. and commonly used reducing agents are sodium hydroxide, ammonium hydroxide, hydrazine hydrate, sodium borohydrate. Once the chemical reaction is over the precipitate is washed with water or acetone and dried in the oven for getting the dry nanoparticles. In the two-step method there is a possibility of agglomeration of the particles in the dispersed fluid. Pak and Cho [68] and Lee [10] used the two-step method for the preparation of Al_2O_3 nanofluid. Xie et al. [92] prepared water and ethylene glycol based Al_2O_3 nanofluids by using the two-step method. Hong [93] produced Fe nanofluids by mixing Fe nanocrystalline powder in ethylene glycol. For the preparation of nanofluids by using the two-step method, the required quantity of nanoparticles can be estimated through the following equation for a given volume concentration.

$$\text{Volume concentration}, \varphi \times 100 = \frac{[W_{particle}/\rho_{particle}]}{[W_{particle}/\rho_{particle}] + [W_{fluid}/\rho_{fluid}]} \quad (1)$$

2.3. Stability of nanofluids

Preparation of stable nanofluids by using the two-step method is a little bit difficult. Nanoparticles are easily settled in the fluids, because of variation of densities between solids and liquids. To overcome the problem of particle sedimentation in the base fluids, several methods like adding surfactants and change of pH of the base fluids are used. Pak and Cho [68] used change of pH of water for the preparation of stable Al_2O_3 and TiO_2 nanofluids. They found that Al_2O_3 nanoparticles are uniformly dispersed in water at a pH of 3 and TiO_2 nanoparticles are uniformly dispersed in water at a pH of 11. Similarly Sundar et al. [30] also found the uniform dispersion of Fe_3O_4 nanoparticles in water at a pH of 3.

Xuan and Li [25] used adding surfactant method by considering few drops of oleic acid for the preparation of stable Cu/oil and Cu/water nanofluids. Murshed et al. [79] also used adding surfactant method by considering Cetyl Trimethyl Ammonium Bromide (C-TAB) surfactant for the preparation of stable Al_2O_3 nanofluid. However, these methods for the preparation of stable nanofluids

Table 2
Available literature on viscosity of nanofluid with various parameters.

Particle	**Size** (nm)	**Base fluid**	**Reference**
AlN	50	EG	Yu et al. [136]
Al_2O_3	28	Water, EG	Wang et al. [72]
Al_2O_3	30	Water	Hwang et al. [31]
Al_2O_3	37	Water	Tseng and Wu [171]
Al_2O_3	28	EG,VPO, EO	Wang et al. [72]
Al_2O_3	13	water	Pak and Cho [68]
Al_2O_3	27, 40 and 50	PG	Prasher et al. [77]
Al_2O_3	20	Water	Mosavian et al. [172]
Al_2O_3	30	Water	Peyghambaradeh et al. [59]
Al_2O_3	36 and 47	Water	Nguyen et al. [121,159]
Al_2O_3	45	60:40% EG/W	Kulkarni et al. [173]
Al_2O_3	10	Water, EG	Lu and Fan [174]
Al_2O_3	80	Water	Murshed et al. [79]
Al_2O_3	40	Decane	Schmidt et al. [149]
Al_2O_3	45 and 150	Water	Anoop et al. [156]
Al_2O_3	<50	Car engine oil	Kole and dey [143]
Al_2O_3	25	CMC	Hojjat et al. [167]
Al_2O_3	43	Water	Chandrasekar et al. [107]
Al_2O_3	95 and 100	EG	Anoop et al. [156]
Al_2O_3	42	Water	Buschmann et al. [165]
Al_2O_3	40	Iso-paraffinic PAO	Schmidt et al. [149]
$BaTiO_3$	580	Ethanol–Isopropanol	Tseng and Lin [183]
$CaCo_3$	20–50	Water	Zhu et al. [153]
CNT	200	Water	Ding et al. [32]
CNT	$L=30\ \mu m,\ d=15$	Water, EG and glycerol	Chen et al. [155]
Cu	200	EG	Garg et al. [175]
Cu	25	Water	Mosavian et al. [172]
CuO	12	EG	Kwak and Kim [74]
CuO	50	Water	Mosavian et al. [172]
CuO	152	EG	Anoop et al. [156]
CuO	50	Base oil	Saeedinia et al. [160]
CuO	29	60:40% EG/W	Namburu et al. [138]
CuO	23–37	Water	Pastoriza-Gallego et al. [157]
CuO	30	60:40% EG/W	Kulkarni et al. [173]
CuO	30–50	CMC	Hojjat et al. [167]
CuO	<50	70:30% PG/W	Naik and Sundar [81]
CuO	29	water	Nguyen et al. [12,159]
CuO	23	EG,VPO, EO	Wang et al. [72]
Fe_2O_3	20	Water	Phuoc and Massoudi [150]
Fe_3O_4	10	Diesel oil, Polydimethylsiloxane	Tsai et al. [176]
Fe_3O_4	11.42	20:80%, 40:60%, 60:40% EG/W	Sundar et al. [146]
Graphite	$\frac{l}{d}\approx 0.02$	ATF, SBO	Yang et al. [24]
MWCNT	20–30	Water	Phuoc et al. [177]
MWCNT	150–200	Water	Amrollahi et al. [181]
Ni	300	Terpineol	Tseng and Chen [117]
SiC	<100	Water	Lee et al. [152]
SiC	170	Water	Yu et al. [164]
Silver	<100	Water	Godson et al. [123]
SiO_2	35, 94 and 190	Ethanol	Chevalier et al. [169]
SiO_2	22	Water	Ferrouillat et al. [161]
SiO_2	15	Synthetic oil	Timofeeva et al. [162]
SiO_2	50	60:40% EG/W	Kulkarni et al. [173]
SiO_2	20, 50 and 100	60:40% EG/W	Namburu et al. [140]
TiO_2	600	EG	Chadwick et al. [73]
TiO_2	20	Water	Tseng and Lin [118]
TiO_2	25	EG	Chen et al. [101]
TiO_2	10	CMC	Hojjat et al. [167]
TiO_2	95,145 and 210	Water	He et al. [168]
TiO_2	21	Water	Boboo et al. [70]
TiO_2	15	Water	Murshed et al. [79]
TiO_2	27	Water	Pak and Cho [68]
TiO_2	25	Water	Chen et al. [78]
TiO_2	21	Water	Duangthongsuk and Wongwises [131]
TiO_2	30	Water	Arani and Amami [133]
TiO_2	21	Water	Turgut et al. [158]
TNT	$L=100,\ d=10$	Water	Chen et al. [178]
TNT	$L=100,\ d=10$	EG	Chen et al. [179]
ZnO	10–20	EG	Yu et al. [163]
ZnO	67.17	EG, glycerol	Mosavi et al. [180]
ZnO	20,40,60	EG	White et al. [61]

can change the thermophysical properties of nanofluids. But these methods are only able to produce stable nanofluids for more than one month. The proper method for the preparation of nanofluids is not established so far. Some more research is needed for obtaining a systematic conclusion in this matter. Synthesis process of various nanofluids and its viscosity enhancement are shown in Table 3.

Table 3
Available literature on synthesis process and viscosity enhancement of nanofluids.

Particle	Size (nm)	Base fluid	Synthesis process	Vol. con. (%)	Enhancement (%)	Reference
AlN	50	EG	Two-step	0.1	1.195	Yu et al. [136]
AlN	50	PG	Two-step	0.1	1.375	Yu et al. [136]
Al_2O_3	< 50	Water	Two-step	0.1–1.0	37–49	Peyghambarzadeh et al. [59]
Al_2O_3	30	Water	Two-step	0.3	2.90	Lee et al. [71]
Al_2O_3	36	water	Two-step	13	210	Nguyen et al. [121,159]
Al_2O_3	47	water	Two-step	13	430	Nguyen et al. [121,159]
Al_2O_3	< 50	Car engine oil	Two-step	1.5	136	Kole and Dey [143]
Al_2O_3	43	water	Two-step	5	136	Chandrasekar et al. [107]
Al_2O_3	27,40 and 50	PG	Two-step	3	29,36,24	Prasher et al. [77]
Al_2O_3	80	water	Two-step	5	82	Murshed et al. [79]
Al_2O_3	28	Water, EG	Two-step	6,3.5	86,39	Wang et al. [72]
Al_2O_3	25	CMC	Two-step	0.2		Hojjat et al. [167]
Al_2O_3	45,150,95and 100	Water	Two-step	8,8,6,6	6,3,77,57	Anoop et al. [156]
$CaCo_3$	50	Water	Two-step	4.11	69	Zhu et al. [153]
Cu	200	EG	Two-step	2	24	Garg et al. [175]
CuO	11,37	Water	Two-step	10 wt%	73,11.5	Pastoriza-Gallego et al. [157]
CuO	152	E G	Two-step	6	32	Anoop et al. [156]
CuO	50	CMC	Two-step	0.2		Hojjat et al. [167]
SiC	< 100	Water	Two-step	3	102	Lee et al. [152]
SiO_2	35,94 and 190	Ethanol	Two-step	5,7,6	95,85,44	Chevalier et al. [169]
TiO_2	15	Water	Two-step	5	86	Murshed et al. [79]
TiO_2	95	Water	Two-step	1.18	11	He et al. [168]
TiO_2	21	Water	Two-step	2	15	Duangthongsuk and Wongwises [131]
TiO_2	10	CMC	Two-step	0.2		Hojjat et al. [167]
TiO_2	21	Water	Two-step	3	135	Turgut et al. [158]

3. Theoretical models

Viscosity is defined as the resistance between two layers of the fluids. Once the nanoparticles are dispersed in the fluids there is a possibility of enhancement in resistance between the two layers of the fluid, if the fluid subjected to shear. It causes enhancement in viscosity of the nanofluid. This enhancement in viscosity of nanofluid can be estimated through solid–fluid homogenous equations. The available theoretical formulas for the estimation of viscosity of nanofluids have been derived from the Einstein [94] model, which is based on the assumption of a linearly viscous fluid containing suspensions of spherical particles.

$$\mu_r = \frac{\mu_{nf}}{\mu_{bf}}(1 + 2.5\,\varphi) \tag{2}$$

where φ and μ are the particle volume concentration and viscosity respectively. The subscripts *bf*, *nf* and *r* refer to the base fluid, nanofluid and the ratio of viscosity of nanofluid to base fluid respectively. The above equation is valid for very low volume concentration ($\varphi \leq 0.02\%$). Although many researchers have contributed to the correction of Einstein's [94] formula based on the assumption of very slow flow, inertial effect in the fluid has been considered negligible by the authors in most of these works, which technically gives linearity to the equations of motion. Brinkman [95] has extended Einstein's [94] formula to a moderate particle volume concentration up to 4%.

$$\frac{\mu_{nf}}{\mu_{bf}} = \left(\frac{1}{(1-\varphi)^{2.5}}\right) \tag{3}$$

Frankel and Acrivos [96] have proposed another type of correlation with the influence of maximum particle volume concentration as follows:

$$\frac{\mu_{nf}}{\mu_{bf}} = 1.125\left[\frac{(\varphi/\varphi_m)^{0.33}}{1-(\varphi/\varphi_m)^{0.33}}\right] \tag{4}$$

where φ_m is the experimental value for the maximum particle volume concentration. Alternatively, Lundgren [97] has offered the subsequent equation as a Taylor series in φ

$$\frac{\mu_{nf}}{\mu_{bf}} = 1 + 2.5\,\varphi + \frac{25}{4}\varphi^2 + 0\,\varphi^3 \tag{5}$$

The above equation is also in the form of the Einstien [94] model, if the term ($0\varphi^3$) or higher is neglected. Batchelor [98] considered the effect due to the Brownian motion of particles for an isotropic suspension of rigid and spherical particles, and proposed

$$\frac{\mu_{nf}}{\mu_{bf}} = 1 + 2.5\,\varphi + 6.5\,\varphi^2 \tag{6}$$

Graham [99] has proposed a generalization form of Eq. (6). His formula, which agrees well with Einstein's [94] model for low value of φ, is as follows:

$$\frac{\mu_{nf}}{\mu_{bf}} = 1 + 2.5\,\varphi + 4.5 \times \left[\frac{1}{(h/d_p)(2+(h/d_p))(1+(h/d_p))^2}\right] \tag{7}$$

where d_p and h are respectively the particle radius and the interparticle spacing.

The above models, Eqs. (4–6), predict the viscosity of nanofluids at very low particle volume concentrations. Most referred model for high particles volume concentrations is given by the Krieger–Dougherty [100] equation.

$$\frac{\mu_{nf}}{\mu_{bf}} = \left(1 - \frac{\varphi_a}{\varphi_m}\right)^{-[\eta]\,\varphi_m} \tag{8}$$

where φ_m is the maximum volume concentration, which varies from 0.495 to 0.54 under quiescent conditions, and is approximately 0.605 at high shear rates, φ_a is the effective volume concentration of aggregates and η is the intrinsic viscosity, whose typical value for mono-disperse suspensions of hard spheres is 2.5. Chen et al. [101] modified the Krieger–Dougherty [100] equation by considering, $\varphi_a = \varphi\left(\frac{a_a}{a}\right)^{3-D}$

$$\frac{\mu_{nf}}{\mu_{bf}} = \left(1 - \frac{\varphi_a}{\varphi_m}\left(\frac{a_a}{a}\right)^{1.2}\right)^{-[\eta]\,\varphi_m} \tag{9}$$

where a_a and a are the radii of aggregates and primary nanoparticles, respectively. The term D is defined as the fractal index,

which for nanoparticles has a typical value of 1.8 (Chen et al. [101]). A simple expression was proposed by Kitano et al. [102] by involving maximum particle volume concentration term φ_m to predict the viscosity of nanofluids:

$$\frac{\mu_{nf}}{\mu_{bf}} = \left(1 - \frac{\varphi_a}{\varphi_m}\right)^{-2} \tag{10}$$

In order to apply Eqs. (8)–(10), φ_m should be calculated.

A generalized equation for the relative elastic moduli of composite materials was proposed by Nielsen [103] for a concentration of dispersed particles.

$$\frac{\mu_{nf}}{\mu_{bf}} = (1 + 1.5\,\varphi_a)e^{(\varphi_a/1-\varphi_m)} \tag{11}$$

Wang et al. [72] developed a model to predict the viscosity of nanofluid as follows:

$$\frac{\mu_{nf}}{\mu_{bf}} = 1 + 7.3\,\varphi + 123\,\varphi^2 \tag{12}$$

Depending on the physical state of the phases, e.g. solid–solid or solid–liquid, different forms of representing concentration are convenient. In a solid–liquid system, the volume fraction of a phase is more usual. In a solid–solid system the Fullman [104] model of the mean free path can also be used.

$$\text{Mean free path is defined as } \lambda = \frac{2}{3} d_p \left(\frac{1-\varphi_p}{\varphi_p}\right) \tag{13}$$

Based on the mean free path λ of the nanoparticles in the base fluid, Neto [105] proposed a correlation, which is given as

$$\frac{\mu_{nf}}{\mu_{bf}} = a\,\frac{1}{\lambda^n} \tag{14}$$

where 'a' and 'n' are constants.

Noni et al. [106] have extended the Neto [105] equation and proposed a viscosity correlation based on the experimental data in the particle volume concentration from 2% to 24% by considering mean free path [104] of alumina (~1.20 μ*m*) and kaolin (~ 3.73 μm) suspended in water. The constants '*b*' and '*n*' in Eq. (15) are 1631 and 2.8 and are obtained by the least square analysis.

$$\frac{\mu_{nf}}{\mu_{bf}} = 1 + b\left(\frac{\varphi}{1-\varphi}\right)^n \tag{15}$$

Chandrasekar et al. [107] have considered Eq. (15) for the estimation of viscosity of Al_2O_3 nanofluid and the coefficients $b=5300$ and $n=2.8$ were obtained by the least square method.

Effective viscosity of particle–fluid mixture in the exponential form has been developed by some researchers. The size of the particles is in the order of micro-size and the empirical equations are given below:

(a) Mooney [108] model is valid in the volume concentration $\varphi_{max}=0.52–0.74$.

$$\mu_m = e\left(\frac{2.5\,\varphi}{1-(\varphi/\varphi_{max})}\right) \tag{16}$$

(b) Thomas and Muthukumar [109]

$$\mu_m = 1 + 2.5\varphi + 10.05\varphi^2 + 0.00273e^{(16.6\varphi)} \tag{17}$$

(c) Metzner [110], maximum particle volume concentration $\varphi_{max}=0.68$

$$\mu_m = \left(1 - \frac{\varphi}{\varphi_{max}}\right)^{-2} \tag{18}$$

(d) Leighton and Acrivos [111], maximum particle concentration $\varphi_{max}=0.58$ and $\mu_{in}=3.0$

$$\mu_m = \left(1 + \frac{0.5\,\mu_{in}\,\varphi}{1-\varphi/\varphi_{max}}\right)^{-2} \tag{19}$$

(e) Barnes et al. [112], maximum particle concentration $\varphi_{max}=0.63–0.71$ and $\mu_{in}=2.71–3.13$

$$\mu_m = \left(1 - \frac{\varphi}{\varphi_{max}}\right)^{-\mu_{in}/\varphi_{max}} \tag{20}$$

(f) Cheng and Law [113]

$$\mu_m = 1 + \frac{5}{2}\varphi + \frac{35}{8}\varphi^2 + \frac{105}{16}\varphi^3 + \frac{1155}{128}\varphi^4 + \frac{3003}{256}\varphi^5 + \ldots \tag{21}$$

$$\begin{aligned}\mu_m = {} & 1 + \frac{5}{2}\varphi + \left(\frac{35}{8} + \frac{5}{4}\beta\right)\varphi^2 + \left(\frac{105}{16} + \frac{35}{8}\beta + \frac{5}{12}\beta^2\right)\varphi^3 \\ & + \left(\frac{1155}{128} + \frac{935}{96}\beta + \frac{235}{96}\beta^2 + \frac{5}{48}\beta^3\right)\varphi^4 \\ & + \left(\frac{3003}{256} + \frac{1125}{64}\beta + \frac{1465}{192}\beta^2 + \frac{95}{96}\beta^3 + \frac{1}{48}\beta^4\right)\varphi^5 + \ldots\end{aligned} \tag{22}$$

where β is called the exponent. If we choose $\beta=2$, we obtain the result very close to the result obtained by the Ward model who suggested the following expression for spherical particles with the experimental data for the concentration up to 35%:

(g) The Ward model cited by Graf [114]:

$$\mu_m = 1 + 2.5\,\varphi_e + (2.5\,\varphi_e)^2 + (2.5\,\varphi_e)^3 + (2.5\,\varphi_e)^4 + \ldots \tag{23}$$

where μ_m is the viscosity of particle–fluid mixture and φ_e is the effective particle concentration.

Avsec and Oblac [115] have calculated the viscosity of nanofluids based on the statistical nanomechanics based on the Cheng and Las [113], and Ward models. They observed that the models predict very good results for the particle–fluid mixture with particle size more than 100 nm. If the particles size is less than 100 nm, those models predict more than 100% deviation compared to the experimental results. Yu and Choi [116] derived the relation with the effect of liquid layer, where monosized spherical particles of radius (r) and particle volume concentration (φ) are suspended in liquid and (h) is the liquid layer thickness and the expression is

$$\varphi_e = \varphi\left(1 + \frac{h}{r}\right)^3 \tag{24}$$

It is noted that for those relationships where the so called maximum concentration is included as a parameter, the effective viscosity approaches infinity when the concentration is equal to the maximum value. This may not be physically reasonable. Strictly speaking, there are only two extreme conditions that are meaningful for the effective viscosity. The first condition is a suspension without particles, implying that the effective viscosity is the same as the base fluid viscosity. The other is a suspension without fluid, which would then theoretically behave as a solid with infinite viscosity.

4. Developed correlations

It is apparent from the above theoretical formulas that the effective viscosity of a viscous fluid containing suspended solid particles is a function only of the base fluid viscosity and the particle volume fraction. In principle, all of these formulas may be used for the determination of the nanofluid viscosity provided that

the linear fluid assumption is satisfied. The limitation and the applicability of such a use are not yet determined. In fact, practically none of the above mentioned models can describe the viscosity of nanofluids exactly in a wide range of the nanoparticle volume fractions. The researchers have developed the viscosity of nanofluid with the effect of volume concentrations and temperatures based on their experimental data in the similar lines of Einstein's [94], Brinkman's [95] and Batchelor's [98] models.

Viscosity correlations by suspending micro-size particles in the fluids by considering an exponential form by Ahuja et al. [4] with particle sizes of 50 μm and 100 μm of polystyrene spheres suspended in aqueous sodium chloride or glycerin are as given below:

$$\frac{\mu_{nf}}{\mu_{bf}} = e^{(2.5\,\varphi/1-1.4\,\varphi)} \tag{25}$$

Tseng and Chen [117] developed viscosity correlation by suspending sub-micrometer nickel powders ($\sim .3\,\mu m$) in terpineol solvent with weight concentration ranging from 0.5% to 10.0% and particle concentration from 3% to 10% with a correlation factor $R^2=0.9952$ which is given as

$$\frac{\mu_{nf}}{\mu_{bf}} = 0.4513\, e^{0.6965\,\varphi} \tag{26}$$

Tseng and Lin [118] developed viscosity correlation by suspending TiO2 nanoparticles (7–20 nm) in distilled water in the particle concentration from 0.05% to 0.12% with a correlation factor $R^2=0.98$ which is given as

$$\frac{\mu_{nf}}{\mu_{bf}} = 13.47\, e^{35.98\,\varphi} \tag{27}$$

Rea et al. [119] developed viscosity correlation for Al_2O_3/water nanofluids by performing a least-curve fitting based on the experimental data of Williams et al. [120] valid up to 6.0% by assuming the exponential form.

$$\frac{\mu_{nf}}{\mu_{bf}} = e^{(4.91\varphi/0.2092-\,\varphi)} \tag{28}$$

Rea et al. [119] also developed viscosity correlation by considering the experimental data of Williams et al. [120] for zirconia/water nanofluids in the volume concentration of 3.0% by assuming the linear form which can be given as

$$\frac{\mu_{nf}}{\mu_{bf}} = 1 + 46.80\varphi + 550.82\varphi^2 \tag{29}$$

Nguyen et al. [121] developed viscosity correlation by dispersing Al_2O_3 (36 nm, 47 nm) and CuO (29 nm) nanoparticles in distilled water in the exponential form and linear assumption of fluid.

$$\left.\begin{aligned}\frac{\mu_{nf}}{\mu_{bf}} &= 0.904\, e^{0.148\,\varphi}\ (47\text{ nm})\\ \frac{\mu_{nf}}{\mu_{bf}} &= 1 + 0.025\varphi + 0.015\varphi^2\ (36\text{ nm})\end{aligned}\right\} Al_2O_3 \text{ nanofluid} \tag{30}$$

$$\left.\frac{\mu_{nf}}{\mu_{bf}} = 1.475-0.319\varphi + 0.051\varphi^2 + 0.009\varphi^3\ (29\text{ nm})\right\} \text{CuO nanofluid} \tag{31}$$

$T = 22\,^\circ C, 1.0\% < \varphi < 13.0\%$

Teipel and Forter-Barth [75] developed viscosity correlation by suspending sub-micrometer aluminum (~30 μm) dispersed in hydroxy-terminated polybutadiene (HTPB) in the concentration $\varphi=50\%$ by assuming the linear viscosity of fluid as follows:

$$\frac{\mu_{suspension}}{\mu_{HTPB}} = 1 + 5.5\,\varphi - 31.4\,\varphi^2 + 74.5\,\varphi^3 \tag{32}$$

Maiga et al. [122] developed viscosity correlations by considering the experimental data of Masuda et al. [8], Lee et al. [10] and Wang et al. [72] for Al_2O_3/water and Al_2O_3/ethylene glycol nanofluids by performing a least-square curve fitting which are given below:

$$\frac{\mu_{nf}}{\mu_{bf}} = 1 + 7.3\,\varphi + 123\,\varphi^2 \quad (Al_2O_3/\text{water}) \tag{33}$$

$$\frac{\mu_{nf}}{\mu_{bf}} = 1-0.19\,\varphi + 306\,\varphi^2 \quad (Al_2O_3/\text{ethylene glycol}) \tag{34}$$

Godson et al. [123] developed viscosity correlation for silver/water nanofluid with the influence of particle volume concentration as follows:

$$\frac{\mu_{nf}}{\mu_{bf}} = 1.005 + 0.497\,\varphi - 0.1149\,\varphi^2 \tag{35}$$

$0.3\% < \varphi < 0.9\%, 50\,^\circ C < T < 90\,^\circ C$

The above equations for viscosity of nanofluids are a function of particle volume concentration. But, practically viscosity of nanofluid also depends on the temperature. Some correlations are available for the estimation of viscosity of nanofluids with the effect of temperature.

The most referred equation on dynamic viscosity of water with the influence of temperature is given by Hagen [124] as follows:

$$\mu_{bf} \times 10^4 = e^{(1.12646-0.039638\,T)/(1-0.00729769T)} \tag{36}$$

where T (K) is the temperature and μ in cP.

The White [125] formula for viscosity of water with the effect of temperature is as follows:

$$\ln\left(\frac{\mu_{base\ fluid}}{\mu_0}\right) \approx a + b\left(\frac{T_0}{T}\right) + c\left(\frac{T_0}{T}\right)^2 \tag{37}$$

where (μ_o, T_o) are the reference values and $a=-2.10,\ b=-4.45,\ c=6.55$

Andrade's equation cited by Reid et al. [126] is an exponential correlation between the viscosity of fluids and their temperature:

$$\mu_{bf} = A\, e^{B/T} \tag{38}$$

where A and B are the functions of volume concentrations.

Yaws [127] presented a viscosity correlation for many industrially important chemical liquids:

$$Log(\mu_{bf}) = A + B\,T^{-1} + C\,T + D\,T^2 \tag{39}$$

where A, B, C and D are the fitting parameters.

Nanofluids with the influence of temperature have been proposed by Nguyen et al. [121] by considering Al_2O_3 (36 nm and 47 nm) and CuO (29 nm) nanofluids as follows:

$$\left.\begin{aligned}\frac{\mu_{nf}}{\mu_{bf}} &= 1.1250-0.007\,T(1.0\%)\\ \frac{\mu_{nf}}{\mu_{bf}} &= 2.1275-0.0215\,T + 0.0002\,T^2\ (4.0\%)\end{aligned}\right\}$$

$$Al_2O_3(36\text{ nm, }47\text{ nm})\text{ and CuO}(29\text{ nm}) \tag{40}$$

$22\,^\circ C < T < 75\,^\circ C, 1.0\% < \varphi < 9.4\%$

Abu-Nada [128] developed another viscosity correlation based on the experimental data of Nguyen et al. [121] for Al_2O_3 nanofluids by considering two dimensional regression analyses as a function of volume concentration and temperature with a correlation factor $R^2=0.998$ as follows:

$$\mu_{nf} = -0.155 - \frac{19.582}{\tilde{T}} + 0.794\,\varphi + \frac{2094.47}{\tilde{T}^2} - 0.192\,\varphi^2 - 8.11\,\frac{\varphi}{\tilde{T}} - \frac{27463.863}{\tilde{T}^3} + 0.0127\,\varphi^3 + 1.6044\,\frac{\varphi^2}{\tilde{T}} + 2.175\,\frac{\varphi}{\tilde{T}^2} \tag{41}$$

Hosseini et al. [129] also developed another viscosity correlation based on the experimental data of Nguyen et al. [121] for Al_2O_3/water nanofluids by considering the least-square regression

technique with the influence of volume concentration, nanoparticle size, effect of the capping layer and temperature.

$$\frac{\mu_{nf}}{\mu_{bf}} = exp\left[m + \alpha\left(\frac{T}{T_0}\right) + \beta(\varphi) + \gamma\left(\frac{d}{1+r}\right)\right] \tag{42}$$

where φ is the percentage of volume concentration 1% and 4%, $m=0.72$, $\alpha=-0.485$, $\beta=14.94$, $\gamma=0.0105$, $T_0=20$ °C and capping layer thickness $r=1$.

Masoumi et al. [130] developed a new viscosity correlation by considering Al2O3 (13 and 28 nm) nanoparticles in water with limited experimental data.

$$\mu_{nf} = \mu_{bf} + \frac{\rho_p V_B d_p^2}{72\,C\delta} \tag{43}$$

$$\delta = \sqrt[3]{\frac{\pi}{6\,\varphi}}d_p,\ V_B = \frac{1}{d_p}\sqrt{\frac{18\,k_b T}{\pi\,\rho_p d_p}} \text{ and } C = \mu_{bf}^{-1}(a\,\varphi + b)$$

The second term in above Eq. (43) is the apparent viscosity arising from the effects of nanoparticles in the fluid and δ is the distance between the centers of the nanoparticles and C is the correction factor.

Duangthongsuk and Wongwises [131] developed a correlation for the estimation of viscosity of TiO_2/water nanofluid in the volume concentration range 0.2% to 2.0% as follows:

$$\frac{\mu_{nf}}{\mu_{bf}} = (a + b\,\varphi + c\,\varphi^2) \tag{44}$$

$$T = 15\ ^\circ\text{C} \Rightarrow a = 1.0226,\ \ b = 0.0477,\ \ c = -0.0112$$

$$T = 25\ ^\circ\text{C} \Rightarrow a = 1.013,\ \ b = 0.0920,\ \ c = -0.015$$

$$T = 35\ ^\circ\text{C} \Rightarrow a = 1.018,\ \ b = 0.112,\ \ c = -0.0177$$

Boboo et al. [70] have proposed the viscosity correlation based on the experimental data valid up to 1.0% volume concentration.

$$\frac{\mu_{nf}}{\mu_{bf}} = 1 + a\,\varphi + b\,\varphi^2 \tag{45}$$

$$a = -0.50437,\ \ b = 1.744\quad (\text{MWCNT/water})$$

$$a = 0.36838,\ \ b = 0.25271\quad (\text{TiO}_2/\text{water})$$

Corcione [132] developed viscosity correlation by considering the various researchers experimental data.

$$\frac{\mu_{nf}}{\mu_{bf}} = \frac{1}{1 - 34.87(d_p/d_f)^{-0.3}(\varphi)^{1.03}} \tag{46}$$

where d_p is the diameter of particle, d_f is the equivalent diameter of the base fluid molecule, $d_f = 0.1(6M/N\,\pi\rho_f)^{1/3}$, M is the molecular weight of the base fluid, ρ_f is the mass density of base fluid at a temperature of 293 K and Avogadro number ($N = 6.022 \times 10^{23}$ mol^{-1}). Arani and Amani [133] used Eq. (46) for the estimation of viscosity of TiO_2/water nanofluid in the volume concentration range from 0.01% to 0.02% in the temperature range from 20 °C to 60 °C.

Vakili-Nezhaad and Dorany [134] proposed viscosity correlation by dispersing single walled carbon nanotubes (SWCNTs) in lubricating oil with the influence of particle volume concentration and temperature.

$$\frac{\mu_{nf}}{\mu_{bf}} = 1 + 1.59\varphi - 16.36\varphi^2 + 50.4\varphi^3\ \ (\text{volume concentration}) \tag{47}$$

$$\frac{\mu_{nf}}{\mu_{bf}} = 0.2\,T^2 - 30.3\,T + 1048\ \ (\text{Temperature}) \tag{48}$$

$$0.01 < \varphi < 0.2\%,\ 25\ ^\circ\text{C} < T < 100\ ^\circ\text{C}$$

Brenner and Condiff [135] have developed a viscosity model for non-spherical particles by considering the shape effects for rod like particles (CNTs).

$$\frac{\mu_{nf}}{\mu_{bf}} = (1 + \eta\,\varphi) \tag{49}$$

$$\eta = \frac{0.312\,r}{\ln(2r - 1.5)} + 2 - \frac{0.5}{\ln(2r - 1.5)} - \frac{1.872}{r}$$

where aspect ratio $r=L/D$, $L=$length of carbon nanotubes and $D=$diameter.

Yu et al. [136] presented a viscosity correlation for the dispersion of 170 nm size SiC particles dispersed in water with effect of temperature and the correlation is valid up to particle volume concentration of 3.7%.

$$\frac{\mu_{nf}}{\mu_{bf}} = 0.00496\,e^{(1736.6/T)} \tag{50}$$

$$0 < \varphi < 3.7\%,\ 25\ ^\circ\text{C} < T < 70\ ^\circ\text{C}$$

where T is the temperature in the unit of Kelvin and μ is the viscosity in centipoise.

Prasher et al. [77] estimated the viscosity of propylene glycol based Al2O3 nanofluid in the volume concentration from 0.5% to 3.0% with the temperature ranging from 30 °C to 60 °C. They observed viscosity enhancement of 4 times more than the thermal conductivity enhancement and also proposed correlation with $C_\mu=10$.

$$\frac{\mu_{nf}}{\mu_{bf}} = 1 + C_\mu\varphi \tag{51}$$

Kulkarni et al. [137] measured the viscosity of CuO (29 nm)/water nanofluid and presented a correlation in the temperature range of 5–50 °C in which constants A and B are the functions of volume concentration.

$$\text{Log}(\mu_{nf}) = A\left(\frac{1}{T}\right) - B \tag{52}$$

Namburu et al. [138] first time prepared 60:40% ethylene glycol/water mixture based CuO nanofluid and also developed correlations. This 60:40% ethylene glycol/water mixture is the mostly used fluid in building heating and cooling and in automobile radiators in cold regions of the world.

$$\text{Log}(\mu_{nf}) = A\,e^{-B\,T} \tag{53}$$

$$A = 1.8375\,(\varphi)^2 - 29.64(\varphi)\ + 165.56(R^2 = 0.9873)$$
$$B = 4 \times 10^{-6}(\varphi)^2 - 0.001(\varphi) + 0.0186(R^2 = 0.9881)$$
$$-35\ ^\circ\text{C} < T < 50\ ^\circ\text{C}\,,\ 1.0\% < \varphi < 6.12\%$$

Namburu et al. [139] also prepared 60:40% ethylene glycol/water mixture based Al2O3 nanofluid and proposed correlation in the similar lines of Eq. (53) with different values of constants A and B.

$$\text{Log}(\mu_{nf}) = A\,e^{-B\,T} \tag{54}$$

$$\left.\begin{aligned} A &= -0.29956\,\varphi^3 + 6.7388\,\varphi^2 - 55.444\,\varphi\ + 236.11\ (\text{R}^2 = 0.9978) \\ B &= \tfrac{(-6.4745\,\varphi^3 + 140.03\,\varphi^2 - 1478.5\,\varphi + 20341)}{10^6}\ (\text{R}^2 = 0.9994) \end{aligned}\right\}$$

$$\text{Al}_2\text{O}_3 \text{ nanofluid} - 35\ ^\circ\text{C} < T < 50\ ^\circ\text{C},\ 1\% < \varphi < 10\%$$

Namburu et al. [140] also prepared 60:40% ethylene glycol/water mixture based SiO2 nanofluid and also proposed correlation similar to Eq. (53).

$$\text{Log}(\mu_{nf}) = Ae^{-BT} \tag{55}$$

$$\left.\begin{aligned} A &= 0.2339\varphi^3 - 3.8943\varphi^2 + 7.1232 + 155.06(R^2 = 0.9904) \\ B &= -7 \times 10^{-6}\varphi^2 - 0.0004\varphi + 0.0192(\,R^2 = 0.9925) \end{aligned}\right\}$$

SiO_2 nanofluid

$-35\,^\circ C < T < 50\,^\circ C, 2 < \varphi < 10\%$

Sahoo et al. [141] extended the experimental data of Namburu et al. [138] by considering 60:40% ethylene glycol/water based Al2O3 nanofluid in the volume concentration range 1.0–10% in the temperature range from −35 °C to 90 °C.

$$\mu_{nf} = Ae^{((B/T)+C\varphi)} \tag{56}$$

$-35\,^\circ C < T < 0^\circ \Rightarrow A = 1.2200 \times 10^{-6},\ B = 4285,$
$C = 0.1448 (R^2 = 0.9984)$

$0\,^\circ C < T < 90^\circ \Rightarrow A = 2.3920 \times 10^{-4},\ B = 2903,$
$C = 0.1265 (R^2 = 0.9958)$

Vajjha and Das [142] carefully analyzed all the experimental data of Namburu et al. [138,139] and Sahoo et al. [141] to develop a general correlation for viscosity of these nanofluids. They derived a correlation which expressed the viscosity in a non-dimensional form, valid for Al_2O_3, CuO and SiO_2 nanofluid in the temperature range $20\,^\circ C < T < 90\,^\circ C$

$$\frac{\mu_{nf}}{\mu_{bf}} = A\, e^{(B\,\varphi)} \tag{57}$$

$0 < \varphi < 0.1\% \Rightarrow A = 0.983,\ B = 12.959$ (Al_2O_3 nanofluid)

$0 < \varphi < 0.06\% \Rightarrow A = 0.9197,\ B = 22.8539$ (CuO nanofluid)

$0 < \varphi < 0.1\% \Rightarrow A = 1.092,\ B = 5.954$ (SiO_2 nanofluid, 20 nm)

$0 < \varphi < 0.1\% \Rightarrow A = 0.9693,\ B = 7.074$ (SiO_2nanofluid, 50 nm)

$0 < \varphi < 0.1\% \Rightarrow A = 1.005,\ B = 4.669$ (SiO_2 nanofluid, 100 nm)

Kole and Dey [143] estimated visocsity of 50:50% propylene glycol/water (car engine coolant) mixture based Al_2O_3 nanofluid in the volume concentration from 0.001% to 0.015% and in the temprature range of 10–50 °C. They used Eq. (53) of Namburu et al. [137] for the estimation of constants A and B with correlation factor R^2=0.99.

$$\mathrm{Log}(\mu_{nf}) = A\, e^{-B\,T} \tag{58}$$

$0.001\% \Rightarrow A = 1.83442,\ B = 0.01345$

$0.004\% \Rightarrow A = 1.88642,\ B = 0.01244$

$0.007\% \Rightarrow A = 1.98529,\ B = 0.01226$

$0.010\% \Rightarrow A = 1.98752,\ B = 0.01128$

$0.015\% \Rightarrow A = 2.13550,\ B = 0.00999$

Chen et al. [101] measured the viscosity of ethylene glycol based TiO_2 nanofluid and proposed correlations with the effect of weight concentration and temperature separately. The correlation for nanofluid with effect of temperature is derived based on the Andrade equation [144] of $\ln\mu = A + \frac{B}{T}$, with a little bit of modification and another parameter C is introduced as follows:

$$\frac{\mu_{nf}}{\mu_{bf}} = 1 + 10.6\varphi + 10.6\varphi^2 \text{ (Effect of weight concentration)} \tag{59}$$

$$\ln\mu_{nf} = A + 1000\frac{B}{(T+C)} \text{ (Effect of temperature)} \tag{60}$$

$0.5\% \Rightarrow A = -3.2114,\ B = 0.86285,\ C = -155.13$

$1.0\% \Rightarrow A = -3.1820,\ B = 0.91603,\ C = -150.35$

$2.0\% \Rightarrow A = -3.3289, B = 0.98375, C = -144.48$

$4.0\% \Rightarrow A = -3.2517,\ B = 0.91226,\ C = -150.74$

$8.0\% \Rightarrow A = -3.7005,\ B = 1.08082,\ C = -138.30$

Kole and Dey [145] measured the viscosity of gear oil (IBP Haulic-68) based CuO nanofluid and developed correlation in the similar lines of Eq. (60) proposed by Chen et al. [101].

$$\ln\mu_{nf} = A + 1000\frac{B}{(T+C)} \tag{61}$$

$0.005\% \Rightarrow A = -0.70784,\ B = 0.70912,\ C = -171.049$

$0.010\% \Rightarrow A = -1.11379,\ B = 1.23013,\ C = -104.976$

$0.015\% \Rightarrow A = -4.94087,\ B = 3.37827,\ C = -26.2139$

$0.020\% \Rightarrow A = -1.10774,\ B = 1.43086,\ C = -87.08024$

$0.025\% \Rightarrow A = -1.61144,\ B = 0.57409,\ C = -160.0491$

Sundar et al. [146] estimated viscosity of 20:80%, 40:60% and 60:40% ethylene glycol/water mixture based magnetic Fe_3O_4 nanofluid and proposed correlations:

$$\frac{\mu_{nf}}{\mu_{bf}} = (1+\varphi)^{0.68} \quad (20:80\% \text{ and } 40:60\% \text{ EG/W}) \tag{62}$$

$$\frac{\mu_{nf}}{\mu_{bf}} = (1+\varphi)^{1.205} \quad (60:40\% \text{ EG/W}) \tag{63}$$

$0 < \varphi < 1.0\%,\ 0\,^\circ C < T < 50\,^\circ C$

Duan et al. [147] developed a viscosity correlation for graphite/water nanofluid by assuming that nanofluid is a pseudoplastic fluid in the volume concentrations range of 1– 4% and R is the shear rate.

$$\frac{\mu_{nf} - \mu_\infty}{\mu_{bf} - \mu_\infty} = \frac{1}{1 + \alpha\, R^n} \tag{64}$$

$1.0\% \Rightarrow \mu_{bf} = 5.02911 \times 10^{10},\ \mu_\infty = 0.0009687,\ \alpha = 3.64600 \times 10^{12},\ n = 0.577278$

$2.0\% \Rightarrow \mu_{bf} = 1.07711 \times 10^{11},\ \mu_\infty = 0.0016582,\ \alpha = 1.81188 \times 10^{12},\ n = 0.814235$

$3.0\% \Rightarrow \mu_{bf} = 4.85370 \times 10^{11},\ \mu_\infty = 0.0021893,\ \alpha = 4.57560 \times 10^{12},\ n = 0.632872$

$4.0\% \Rightarrow \mu_{bf} = 1.11869 \times 10^{13},\ \mu_\infty = 0.0025989,\ \alpha = 5.03442 \times 10^{13},\ n = 0.689077$

Naik and Sundar [81] developed a simple viscosity correlation for 30:70% propylene glycol/water mixture based CuO nanofluid with the effect of particle volume concentration and temperature.

$$\frac{\mu_{nf}}{\mu_{bf}} = 3.444\left(\frac{T_{max}}{T_{min}}\right)^{0.514} \varphi^{0.1829} \tag{65}$$

$T_{max} = 60\,^\circ C,\ T_{min} = 5\,^\circ C\ 0 < \varphi < 1.2\%$

Experimental correlations for the viscosity of nanofluids proposed by various researchers are summarized in Table 4.

5. Experimental investigations

The theoretical models for viscosity of nanofluids are suitable only for very low volume concentration. These correlations failed

Table 4
Available correlations in the literature for viscosity of different kinds of nanofluids with different volume concentrations.

Correlation	Particle	Base fluid	Vol. con. (%)	Comment		Reference
				T	**V**	
$Log(\mu_{nf})=Ae^{-BT}$	Al_2O_3	60:40% EG/W	1.0–10	√	√	Namburu et al. [139]
$\frac{\mu_{nf}}{\mu_{bf}}=1.1250-0.007\,T(1.0\%)$	Al_2O_3 and CuO	Water	1.0–9.4	√	—	Nguyen et al. [121]
$\frac{\mu_{nf}}{\mu_{bf}}=2.1275-0.0215\,T+0.0002\,T^2(4.0\%)$						
$\frac{\mu_{nf}}{\mu_{bf}}=1+7.3\,\varphi+123\,\varphi^2$	Al_2O_3	Water	0.0–6.0	—	√	Maiga et al. [122]
$\frac{\mu_{suspension}}{\mu_{HTPB}}=1+5.5\varphi-31.4\varphi^2+74.5\,\varphi^3$	Al_2O_3	HTPB	50.0	—	√	Teipel and Forter-Barth [75]
$\frac{\mu_{nf}}{\mu_{bf}}=1-0.19\varphi+306\varphi^2$	Al_2O_3	EG		—	√	Maiga et al. [122]
$\frac{\mu_{nf}}{\mu_{bf}}=0.904\,e^{0.148\,\varphi}(47\text{ nm})$	Al_2O_3	Water	1.0–13.0	√	—	Nguyen et al. [121]
$\frac{\mu_{nf}}{\mu_{bf}}=1+0.025\varphi+0.015\varphi^2(36\text{ nm})$				—	√	
$\frac{\mu_{nf}}{\mu_{bf}}=e^{(4.91\varphi/0.2092-\varphi)}$	Al_2O_3	Water	0.0–6.0	√	—	Rea et al. [119]
$\frac{\mu_{nf}}{\mu_{bf}}=1+C_\mu\varphi$	Al_2O_3	PG	0.5–3.0	√	—	Prasher et al. [77]
$\mu_{nf}=Ae^{((B/T)+C_\varphi)}$	Al_2O_3	60:40% EG/W	1.0–10.0	√	—	Sahoo et al. [141]
$Log(\mu_{nf})=A\,e^{-BT}$	Al_2O_3	50:50% PG/W	0.001–0.015	√	—	Kole and Dey [143]
$\frac{\mu_{nf}}{\mu_{bf}}=A\,e^{(B\varphi)}$	Al_2O_3, CuO and SiO_2	60:40% EG/W	0.0–0.1, 0.0–0.06, 0.0–0.1	√	—	Vajjha and Das [142]
$\frac{\mu_{nf}}{\mu_{bf}}=(1+\eta\varphi)$	CNT	Water		√	—	Brenner and Condiff [135]
$\frac{\mu_{nf}}{\mu_{bf}}=3.444\left(\frac{T_{max}}{T_{min}}\right)^{0.514}\varphi^{0.1829}$	CuO	30:70% PG/W	0.0–1.2	√	—	Naik and Sundar [81]
$\ln\mu_{nf}=A+1000.\frac{B}{(T+C)}$	CuO	Gear oil (IBP Haulic-68)	0.005–0.025	√	—	Kole and Dey [145]
$Log(\mu_{nf})=A\,e^{-BT}$	CuO	60:40%	1.0–6.12	√	—	Namburu et al. [138]
$Log(\mu_{nf})=A(\frac{1}{T})-B$	CuO	Water	1.0–6.12	√	—	Kulkarni et al. [137]
$\frac{\mu_{nf}}{\mu_{bf}}=1.475-0.319\varphi+0.051\varphi^2+0.009\varphi^3$	CuO	Water	1.0–13.0	—	√	Nguyen et al. [121]
$\frac{\mu_{nf}-\mu_\infty}{\mu_{bf}-\mu_\infty}=\frac{1}{1+\alpha R^n}$	Graphene	Water	1.0–4.0	√	—	Duan et al. [147]
$\frac{\mu_{nf}}{\mu_{bf}}=(1+\varphi)^{0.68}$	Fe_3O_4	20:80%, 40:60% EG/W	0.0–1.0	√	√	Sundar et al. [146]
$\frac{\mu_{nf}}{\mu_{bf}}=(1+\varphi)^{1.205}$	Fe_3O_4	60:40% EG/W	0.0–1.0	√	√	Sundar et al. [146]
$\frac{\mu_{nf}}{\mu_{bf}}=1+a\varphi+b\varphi^2\ \ a=-0.50437,\ b=1.744$	MWCNT	Water	0.0–1.0	—	√	Boboo et al. [70]
$\frac{\mu_{nf}}{\mu_{bf}}=0.4513\,e^{0.6965\varphi}$	Nickel	Terpineol	0.5–10	√	—	Tseng and Chen [117]
$\frac{\mu_{nf}}{\mu_{bf}}=1+1.59\varphi-16.36\varphi^2+50.4\varphi^3$	SWCNT	Lubricating oil	0.01–0.2	—	√	Vakili-Nezhaad and Dorany [134]
$\frac{\mu_{nf}}{\mu_{bf}}=0.2T^2-30.3T+1048$	SWCNT	Lubricating oil	0.01–0.2	√	—	Vakili-Nezhaad and Dorany [134]
$\frac{\mu_{nf}}{\mu_{bf}}=e^{(2.5\varphi/1-1.4\varphi)}$	Polystyrene spheres	Sodium chloride or glycerin	0–8.2	√	—	Ahuja et al. [4]
$\frac{\mu_{nf}}{\mu_{bf}}=1.005+0.497\varphi-0.1149\varphi^2$	Silver	Water	0.3–0.9	—	√	Godson et al. [123]
$\frac{\mu_{nf}}{\mu_{bf}}=0.00496e^{(1736.6/T)}$	SiC	Water	0.0–3.7	√	—	Yu et al. [136]
$Log(\mu_{nf})=A\,e^{-BT}$	SiO_2	60:40% EG/W	2.0–10.0	√	√	Namburu et al. [140]
$\frac{\mu_{nf}}{\mu_{bf}}=1+10.6\varphi+10.6\varphi^2$	TiO_2	EG	0.5–8.0	—	√	Chen et al. [101]
$\ln\mu_{nf}=A+1000.\frac{B}{(T+C)}$	TiO_2	EG	0.5–8.0	√	—	Chen et al. [101]
$\frac{\mu_{nf}}{\mu_{bf}}=1+a\varphi+b\varphi^2\ \ a=0.36838,\ b=0.25271$	TiO_2	Water	0.0–1.0	—	√	Boboo et al. [70]
$\frac{\mu_{nf}}{\mu_{bf}}=(a+b\varphi+c\varphi^2)$	TiO_2	Water	0.2–2.0	—	√	Duangthongsuk and Wongwises [131]
$\frac{\mu_{nf}}{\mu_{bf}}=13.47\,e^{35.98\varphi}$	TiO_2	Water	0.05–0.12	√	—	Tseng and Lin [118]
$\frac{\mu_{nf}}{\mu_{bf}}=1+46.80\varphi+550.82\varphi^2$	Zirconia	Water	0.0–3.0	—	√	Rea et al. [119]

Note: EG—ethylene glycol, PG—propylene glycol, CMC—carboxymethlecellulose, ATF—automatic transmission fluid, SBO—synthetic base oil, VPO—vaccum pump oil, EO—engine oil, EG/W—ethylene glycol and water mixture, HTPB—hydroxy-terminated polybutadiene, *T*—temperature, *V*—volume concentration.

to predict the nanofluid viscosity for higher volume concentrations. The models are derived based on linear assumptions of viscosity of base fluid and it is a function of base fluid viscosity and volume concentration. In actual practice viscosity of base fluid also depends on the temperature. The viscosity models failed to predict the viscosity of nanofluid with the influence of temperature. For fully understanding the viscosity of nanofluids with the influence of volume concentration, temperature, particle size and base fluid an experimental investigation is needed.

5.1. *Effect of concentration*

Viscosity of nanofluid increases with the increase of particle volume concentration in the base fluid. Initial viscosity measurements with the dispersion of micrometer sized polystyrene spheres in aqueous sodium chloride were performed by Ahuja [4] who found better viscosity enhancement compared to base fluid. After the invention of nanometer sized particles first time Masuda et al. [8] measured the viscosity of $A1_2O_3$, SiO_2, and TiO_2 ultrafine particles dispersed in water and found better viscosity enhancement. Pak and Choi [68] measured the viscosity of Al_2O_3 and TiO_2 nanofluids in the volume concentration from 1.0% to 10% and found viscosity enhancement. Kwak and Kim [74] measured the viscosity of CuO/ethylene glycol nanofluid and obtained viscosity enhancement with the addition of nanoparticles. Chen et al. [78] measured the rheological behavior of titanate nanotubes (TNTs) dispersed in ethylene glycol and found that the viscosity enhancements of various particle volume concentrations of 0.5%, 1.0%, 2.0%, 4.0% and 0.8% are 3.3%, 7%, 16.22%,

26.34% and 70.96% respectively. Chandrasekar et al. [107] obtained a maximum viscosity enhancement of 136% at 5% volume concentration of Al_2O_3/water nanofluid. Nguyen et al. [121] obtained 210% viscosity enhancement with 13% volume concentration of Al_2O_3/water nanofluid and also proposed a viscosity correlation with the effect of particles concentration and temperature. Das et al. [83] and Putra et al. [148] have obtained the Newtonian behavior of Al_2O_3/water nanofluid in the measured volume concentration of 4% and found viscosity increases with the increase of particle concentration. Masuda et al. [8] obtained 86% viscosity enhancement with 5.0% volume concentration of TiO_2/water nanofluid. Boboo et al. [70] obtained visocsity enhnacment of 12.9% for SWCNT/water and 6.8% viscosity enhancement for TiO_2/water nanofluid at 1.0% volume concentration compared to base fluid. Lee et al. [10] observed viscosity enhancement of 2.90% for Al_2O_3/water nanofluid at 0.3% at 21 °C temperature. Schmidt et al. [149] measured the viscosity of Al_2O_3 nanoparticles dispersed in decane and isoparaffinic polyalphaolefin (PAO) in the volume concentration of 0.25–1.0%. Phuoc and Massoudi [150] measured the viscosity of Fe_2O_3/water nanofluid by considering polyvinylpyrrolidone (PVP) or polyethylene oxide (PEO) as a surfactant and they observed that Fe_2O_3 nanofluid with 0.2% PVP has Newtonian behavior up to $\varphi < 0.02\%$ and as the concentration increases more than 0.02% it shows non-Newtonian behavior and similar results were also observed with 0.2% PEO surfactant. Anoop et al. [151] observed maximum viscosity enhancement of 32% with 6.0% volume concentration of CuO/ethylene glycol nanofluid. Kole and Dey [143] observed maximum viscosity enhancement of 136% with 1.5% volume concentration of Al_2O_3/car engine oil nanofluid. Lee et al. [152] observed maximum viscosity enhancement of 102% with 3.0% volume concentration of SiC/water nanofluid. Zhu et al. [153] obtained 69% viscosity enhancement with 4.11% of $CaCO_3$/water nanofluid. Sundar et al. [146] observed maximum viscosity enhancement of 294% with 1.0% volume concentration of Fe_3O_4 nanoparticles dispersed in 60:40% ethylene glycol/water mixture. Wang et al. [72] observed that the effective viscosity of nanofluid containing 5% volume concentration of Al_2O_3 nanoparticles in distilled water was prepared by mechanical blending technique and 86% enhancement was observed. They also observed 40% enhancement in viscosity of ethylene glycol in the volume concentration of 3.5% of Al_2O_3 nanoparticles. Their results state that viscosity of nanofluid also depends on the dispersion method. Duan et al. [154] measured the viscosity of Al_2O_3/water nanofluid in the volume concentrations of 1%, 2%, 3%, 4% and 5% and observed that the nanofluid behaves like a non-Newtonian fluid.

Nanofluid viscosity for Al_2O_3 and TiO_2 is broadly studied and all the researchers mostly explained the similar trend of viscosity enhancement with the increase of particle volume concentration. Viscosity of Al_2O_3 nanofluid with the effect of particle volume concentration is shown in Fig. 1.

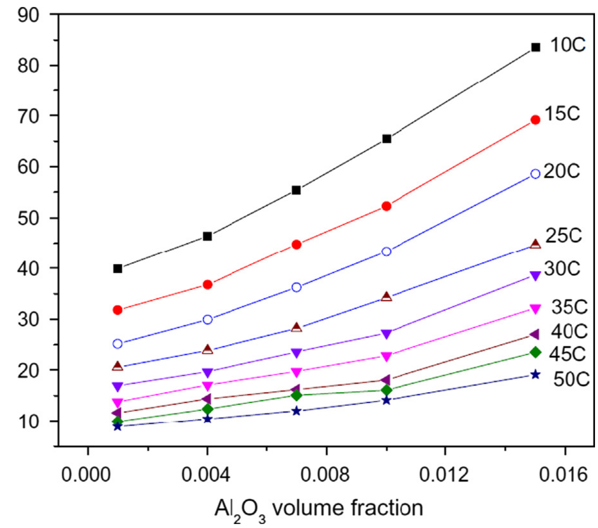

Fig. 1. Viscosity of Al_2O_3 nanofluid with effect of particle volume concentration (Kole and Dey [143]).

5.2. Effect of temperature

Viscosity of nanofluid with the change of temperature is also very important, because for commercial applications of nanofluids in heat transfer equipments viscosity at different temperatures is required. In this regard, some researchers have explained the viscosity of nanofluid with the effect of temperature. Godson et al. [123] measured the viscosity of silver/water nanofluid in the volume concentrations from 0.3% to 0.9% between the temperatures of 50 °C and 90 °C and found 1.45 times viscosity enhancement for 0.9% volume concentration. Duangthongsuk and Wongwises [131] have obtained 4–15% viscosity enhancement with TiO_2/water nanofluid in the volume concentrations of 0.2–2.0% and in the temperature range of 15–35 °C. Peyghambarzadeh et al. [59] measured the viscosity of Al_2O_3/water nanofluid in the volume concentration of 0.15–1.0% in the temperature range from 37 °C to 49 °C. Chen et al. [155] found the viscosity ratios of CNT nanofluid to the corresponding value of the base fluid. Their results indicate that, at low volume fractions $\varphi < 0.004$, nanofluids have lower viscosity than the corresponding base fluids due to the lubricating effect of nanoparticles. When the volume fraction is higher than 0.004, the viscosity increases with nanoparticle loadings. When the temperature is higher than 55 °C it appears to increase substantially with the temperature. Anoop et al. [156] measured the viscosity of CuO/ethylene glycol, Al_2O_3/ethylene glycol and Al_2O_3/water nanofluids in the temperature range from 20 °C to 50 °C and found for all the nanofluids viscosity decreases with the increase of temperature. They also explained that viscosity ratio for water based nanofluid is more compared to viscosity ratio of ethylene glycol based nanofluids. Yang et al. [24] experimentally measured kinematic viscosity of graphite/water nanofluid in the temperatures of 35 °C, 43 °C, 50 °C and 70 °C and obtained 44.8 cst enhancement at a temperature of 35 °C and 14.5 cst at a temperature of 70 °C at 2% wt concentration. Lee et al. [152] measured viscosity of SiC/water nanofluid in the volume concentration range from 0.0001% to 3.0% in the temperature range from 28 °C to 72 °C and found viscosity enhancement with the addition of nanoparticles. Pastoriza-Gallego et al. [157] investigated the viscosity of CuO/water nanofluid in the temperature range from 283.15 K to 323.15 K and they also reported that viscosity decreases with increase of temperature at 323.15 K. Masuda et al. [8] have measured for the first time the viscosity of water based nanofluid in the temperature range from 273 K to 340 K. Turgut et al. [158] measured the viscosity of TiO_2/water nanofluid in the temperature range of 13–55 °C in the volume concentration range of 0.2–3.0% and found a decrease of viscosity with the increase of temperature. Nguyen et al. [121,159] measured the viscosity of Al_2O_3/water and CuO/water nanofluid in the temperature range from 21 °C to 75 °C and found that the viscosity of nanofluids decreases with the increase of temperature.

Namburu et al. [138] prepared for the first time 60:40% ethylene glycol/water mixture based CuO nanofluid and also developed correlations. This 60:40% ethylene glycol/water mixture is the most used fluid in building heating and cooling and in automobile radiators in cold regions of the world. They also studied the viscosity of CuO nanofluid in the temperature range of −35 °C to 50 °C and found viscosity of nanofluid decreases exponentially with the increase of temperature. In another subsequent studies of Namburu et al. [139,140] for Al_2O_3 and SiO_2

nanoparticles dispersed in 60:40% ethylene glycol/water mixture in the temperature range −35 °C to 50 °C they observed all the fluids have Newtonian behavior in the tested volume concentration range. Sahoo et al. [141] extended the experimental data of Namburu et al. [138] by considering 60:40% ethylene glycol/water based Al2O3 nanofluid in the volume concentration range 1.0–10% in the temperature range of −35 °C to 90 °C. Naik and Sundar [81] experimentally determined the viscosity of 30:70% propylene glycol/water mixture based CuO nanofluid and observed that viscosity decreases exponentially with the increase of temperature from 5 °C to 60 °C. Kole and Dey [143] measured viscosity of 50:50% (car engine coolant) propylene glycol/water mixture based Al_2O_3 nanofluid in the temperature range from 10 °C to 50 °C and proposed a correlation in a similar way to Namburu et al. [138]. Another study of Kole and Dey [145] measured viscosity of gear oil based CuO nanofluid in the temperature range from 10 °C to 80 °C and observed non-Newtonian behavior with the addition of CuO nanoparticle to gear oil. Saeedinia et al. [160] considered CuO/base oil nanofluid in the temperature up to 70 °C, Ding et al. [32] considered CNT/water nanofluid up to 40 °C, Ferrouilllat et al. [161] considered SiO_2/water nanofluid up to 20 °C–70 °C, Timofeeva et al. [162] considered SiO_2/synthetic oil (Therminol 66) in the temperature range 15 °C–135 °C, Yu et al. [163] considered ZnO/EG nanofluid in the temperature range of 20 °C–60 °C, Yu et al. [164] considered SiC/water nanofluid in the temperature range of 25–70 °C, Buschman et al. [165] considered ceramic/water nanofluid in the temperature up to 60 °C and Williams et al. [120] considered zirconia/water and alumina/water nanofluid in the temperature range from 20 °C to 80 °C by measuring the viscosity with effect of temperatures. All the above researchers found the similar trend of decrease in viscosity of nanofluids with the increase of temperatures. Aladag et al. [166] measured the viscosity and shearing time on viscosity for Al_2O_3/water and CNT/water based nanofluids at low concentration and low temperatures from 2 °C to 10 °C with stress-controlled rheometer equipped with parallel plate geometry under up and down shear stress ramp. Viscosity of CuO nanofluid with the effect of temperature is shown in Fig. 2.

5.3. Effect of particle size and shape

Viscosity of nanofluid increases with the increase of particle concentration. The effect of particle size is also very important on viscosity of nanofluid. Nguyen et al. [121] measured the viscosity of Al_2O_3/water nanofluid with the effect of particle size. They considered two particle sizes of 36 and 47 nm of Al_2O_3 and observed that both the nanofluids below 4% particle volume concentration are exhibiting the same results; for higher particle volume concentration, viscosity of 36 nm size particle is less than the viscosity of 47 nm size particle. Prasher et al. [77] prepared PG based Al_2O_3 nanofluid with the particle sizes of 27, 40 and 50 nm and observed 29%, 36% and 24% viscosity enhancement for 3% volume concentration respectively. Anoop et al. [156] prepared water based Al_2O_3 nanofluid with particle sizes of 95, 100 and 150 nm in the particle weight concentrations of 1%, 2%, 4% and 6% and explained that the viscosity of nanofluid increases with the decrease of particle size. Hojjat et al. [167] prepared three different nanofluids by dispersing Al_2O_3 (25 nm), CuO (30–50 nm) and TiO_2 (10 nm) nanoparticles in aqueous solution of carboxymethylcellulose for the estimation of viscosity. They observed that base fluid and all nanofluids exhibited shear-thinning rheological behavior at 0.2% volume concentration at the temperature of 25 °C and they also observed that Al_2O_3 nanofluid viscosity is more than CuO and TiO_2 nanofluids viscosity. Murshed et al. [79] considered 80 nm size of Al_2O_3 and 15 nm size of TiO_2 nanoaprticles dispersed in water for the preparation of nanofluids and found maximum of 82% with Al_2O_3/water nanofluid and 86% with TiO_2/water nanofluid at 5% volume concentration. He et al. [168] measured the viscosity of TiO_2/water nanofluid with the particle sizes of 95, 145 and 210 nm and they observed a decrease in viscosity with the increase in particle size. Chevalier et al. [169] measured the viscosity of SiO_2/ethanol nanofluid with the particle sizes of 35, 94 and 190 nm in the particle concentration range of 1.4–7% and discovered that viscosity increases with the decrease of particle size. Namburu et al. [140] measured the viscosity of different sizes of 20, 50, 100 nm SiO_2 nanoparticles dispersed in 60:40% ethylene glycol and water mixture and found that viscosity decreases with the increase of particle size. Pastoriza-Gallego et al. [157] investigated the viscosity of CuO/water nanofluid with 11 and 23 nm particles size and found maximum of 73% and 11.5% enhancement at weight concentration of 10% respectively. Timofeeva et al. [170] explained that the viscosity of nanofluid is strongly dependent on the particle shape and they found higher results with elongated particles like platelets and cylinders compared to spherical particles. Viscosity of Al_2O_3 nanofluid with the effect of particle size is shown in Fig. 3.

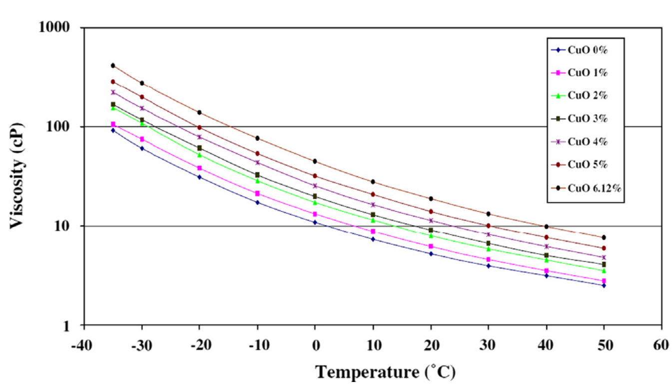

Fig. 2. Viscosity of CuO nanofluid with effect of temperature (Namburu et al. [138]).

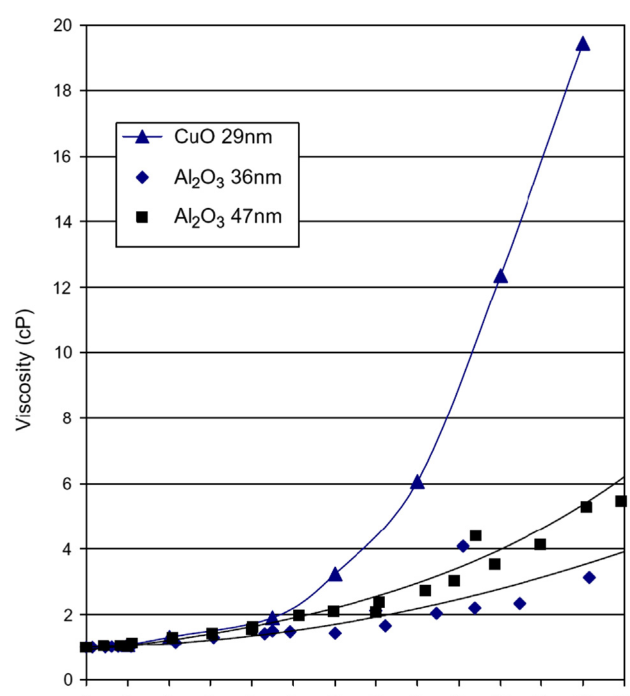

Fig. 3. Viscosity of Al_2O_3 nanofluid with effect of particle size (Nguyen et al. [121]).

5.4. Effect of base fluid

Viscosity of nanofluid is purely dependent on the viscosity of base fluid. Some researcher used water, ethylene glycol, propylene glycol, mixture of ethylene glycol/water and mixture of propylene glycol/water for the preparation of nanofluids and explained that nanofluid viscosity is strongly dependent on the base fluid viscosity. Chen et al. [155] prepared MWCNTs nanofluids by considering water, ethylene glycol, glycerol and silicone oil as base fluids and observed that ethylene glycol and glycerin based nanofluids diminish the viscosity enhancement when the temperature is higher than 55 °C. Chen et al. [101] prepared TiO_2 nanofluids by considering EG and water as base fluids and they found maximum viscosity enhancement of 23% with 1.86% volume concentration of TiO_2/EG nanofluid and maximum viscosity enhancement of 11% with 1.2% volume concentration of TiO_2/water nanofluid. Chen et al. [78] prepared TNT nanofluids by considering water and EG as base fluids and obtained viscosity enhancement of 70.96% at 1.86% volume concentration of TNT/water nanofluid and viscosity enhancement of 82% at 0.6% volume concentration of TNT/EG nanofluid. Wang et al. [72] considered water and EG as a base fluid for the preparation of Al_2O_3 nanofluids. They found the maximum viscosity enhancement of 86% at 6.0% volume concentration of Al_2O_3/water and viscosity enhancement of 39% at 3.5% volume concentration of Al_2O_3/EG as base fluids. Yu et al. [136] considered aluminum nitride nanoparticles dispersed in EG and PG in order to study the viscosity of nanofluid with the effect of base fluids. They measured the viscosity in the particle volume concentration of 0.1% and found 1.195% enhancement with EG and 1.375% enhancement with PG used as a base fluid. Sundar et al. [146] considered three types of base fluids like 20:80%, 40:60% and 60:40% EG/W mixtures for the preparation of magnetic Fe_3O_4 nanofluids. They found 296% viscosity enhancement with 60:40% EG/W mixture based nanofluid compared to other nanofluids and also explained that the viscosity of nanofluid is strongly dependent on the viscosity of base fluid. Viscosity of Fe_3O_4 nanofluid with the effect of base fluid is shown in Fig. 4.

6. Conclusions

In this review an attempt has been made to understand the viscosity of nanofluids both experimentally and theoretically

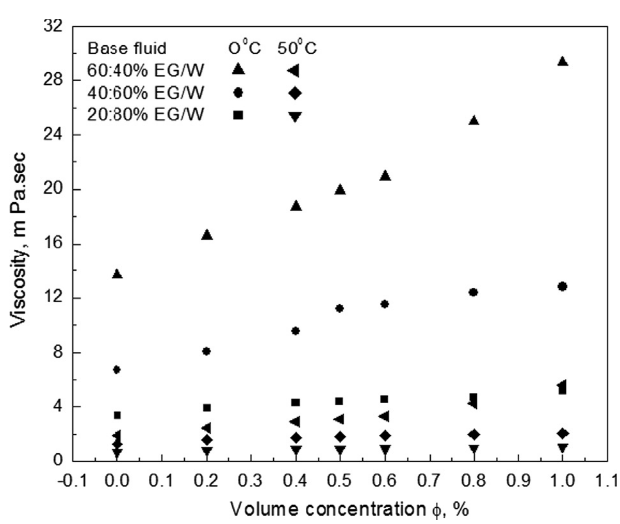

Fig. 4. Viscosity of Fe_3O_4 nanofluid with effect of base fluid (Sundar et al. [146]).

through the available literature. The review is also extended to the viscosity of nanofluids with the effect of particle volume concentration, temperature, particle size and shape and base fluid. Most of the investigators obtained viscosity enhancement with the dispersion of nanoparticles in the base fluid and further enhancement was also obtained with the increase of particle concentration. Viscosity of nanofluid decreases with the increase of temperature. Empirical correlations are available to estimate the viscosity of nanofluids with the effect of particle volume concentration and temperature. Some few correlations are available for the estimation of viscosity with the effect of particle size. Particle size is also an important parameter for the estimation of viscosity of nanofluids. No exact theoretical mechanism or empirical correlations are available for the estimation of viscosity of nanofluids.

For the practical application of nanofluids in mechanical devices like heat exchangers and condensers, it is very essential to study the viscosity and rheological behavior of nanofluids. There is a small deviation in the published results. Researchers reported that nanofluids exhibit Newtonian behavior [10,72,76–78,101,118,138,143,156,160,166,169] and some researchers reported nanofluids exhibit non-Newtonian behavior [167,100,166,9,78,75,147].

Before measuring the viscosity of nanofluids make sure that particles are uniformly dispersed in the base fluid. The viscosity of Al_2O_3/water nanofluids was measured by various researchers like Hojjat et al. [167], Lee et al. [71], Hwang et al. [31], Chandasekar et al. [107], Anoop et al. [156], Murshed et al. [79], Pak and Cho [68] and Wang et al. [72]. With the use of the same nanoparticles and base fluid, they obtained different percentages of viscosity enhancement. This is caused due to various particles size, volume concentrations and different methods of the preparation of nanofluid. In a similar way, the viscosity of TiO_2/water nanofluid wasmeasured by Murshed et al. [79] and Chen et al. [101] and they obtained different viscosity enhancements. Because of various nanoparticle sizes they obtained different viscosity enhancements. With the same nanofluids and same volume concentrations researchers published different viscosity enhancements. There is a lot of discussion on the viscosity of nanofluids with the effect of particle size. Some researchers reported that there is no significant effect on viscosity with the particle size [77], but most of the researchers reported that particle size and shape are also important to determine nanofluid viscosity. Most of the researchers expressed that the viscosity of nanofluid increases with the decrease of particle size [169,138,156,157,174] and some expressed that viscosity decreases with the increase of particle size [77,121,159,168].

Viscosity of nanofluid increases linearly with the increase of volume concentration which has been obtained by Nguyen et al. [121] and Maiga et al. [122] for Al_2O_3/water, Duanthongsuk and Wongwises [131] for TiO_2/water, Godson et al. [123] for silver/water, Rea et al. [119] for zirconia/water, Boboo et al. [70] for MWCNT/water and Vikili-Nezhaad and Dorang [134] for SMCNT/lube oil. Some other researchers like Namburu et al. [139] by dispersing Al_2O_3, CuO and SiO_2 in 60:40% ethylene glycol and water mixture, Tseng and Chen [117] in Ni/terpineol and Sundar et al. [146] by dispersing Fe_3O_4 in 20:80%, 40:60% and 60:40% ethylene glycol and water mixture obtained that the viscosity of nanofluid increases non-linearly with the increase of volume concentration. For viscosities of graphite based nanofluid Dung et al. [147] estimated and observed shear thinning non-Newtonian behavior. A benchmark study on the viscosity of different kinds of nanofluids was performed by Venerus et al. [182] for heat transfer applications.

There is no common empirical correlation and theoretical model for the estimation of viscosity of all nanofluids with effect of particle concentration, size and temperature. With this result more investigations are needed for the viscosity of nanofluids to

develop a common empirical equation and also further investigations would be helpful to obtain more reliable data for the upcoming nanofluid applications.

Acknowledgment

The authors would like to thank the Portuguese Foundation of Cinecia e Technologia, through a grant funded by Ministry of Science and Technology. One of the authors (L.S.S.) would like to thank FCT for his Post-Doctoral research grant (SFRH/BPD/79104/2011).

References

[1] Maxwell JC. A treatise on electricity and magnetism. 2nd ed. Clarendon Press; 435.
[2] Hamilton RL, Crasser OK. Thermal conductivity of heterogeneous two-component systems. Industrial and Engineering Chemistry Fundamentals 1962;1:187–91.
[3] Akoh H, Tsukasaki Y, Yatsuya S, Tasaki A. Magnetic properties of ferromagnetic ultrafine particles prepared by a vacuum evaporation on running oil substrate. Journal of Crystal Growth 1978;45:495–500.
[4] Ahuja AS. Augmentation of heat transport in laminar flow of polystyrene suspension. I. Experimental and results. Journal of Applied Physics 1975;46:3408–16.
[5] Choi SUS, Tran TN. Experimental studies of the effects of non-Newtonian surfactant solutions on the performance of a shell-and-tube heat exchanger. In: Siginer DA, Dhaubhadel MN, editors. Recent developments in non-Newtonian flows and industrial applications, 124. New York: ASME; 1991. p. 47–52.
[6] Choi SUS, Cho YI, Kasza KE. Degradation effects of dilute polymer solutions on turbulent friction and heat transfer behaviour. Journal of Non-Newtonian Fluid Mechanics 1992;41:289–307.
[7] Choi SUS, France DM, Knodel BD. Impact of advanced fluids on costs of district cooling systems. Proceedings of the 83rd Annual International District Heating and Cooling Association Conference, Danvers, MA, June 13–17. The International District Heating and Cooling Association.343–59.
[8] Masuda H, Ebata A, Teramae K, Hishinuma N. Alteration of thermal conductivity and viscosity of liquid by dispersing ultra-fine particles (dispersion of y-$A1_2O_3$, SiO_2, and TiO_2 ultra-fine particles). Netsu Bussei (Japan) 1993;4:227–33.
[9] Choi SUS. Enhancing thermal conductivity of fluids with nanoparticles. In: Siginer DA, Wang HP, editors. Developments and applications of non-Newtonian flows, 66. New York: ASME; 1995. p. 99 FED 231/MD.
[10] Lee S, Choi SUS, Li S, Eastman JA. Measuring thermal conductivity of fluids containing oxide nanoparticles. Journal of Heat Transfer 1999;121:280–90.
[11] Choi SUS, Zhang ZG, Yu W, Lockwood FE, Grulke EA. Anomalous thermal conductivity enhancement in nanotube suspensions. Applied Physics Letters 2001;79:2252.
[12] Heris SZ, S.Gh. Etemad, Esfahany MN. Experimental investigation of oxide nanofluids laminar flow convective heat transfer. International Communications in Heat and Mass Transfer 2006;33:529–35.
[13] Eastman JA, Choi SUS, Li S, Yu W, Thompson LJ. Anomalously increased effective thermal conductivities of ethylene glycol-based nanofluids containing copper nanoparticles. Applied Physics Letters 2001;78:718–20.
[14] Li CH, Peterson GP. Experimental investigation of temperature and volume fraction variations on the effective thermal conductivity of nanoparticle suspensions (nanofluids). Journal of Applied Physics 2006;99:084314–21.
[15] Philip J, Shima D, Raj B. Enhancement of thermal conductivity in magnetite based nanofluid due to chainlike structures. Applied Physics Letters 2007;91:203108–11.
[16] Karthikeyan NR, Philip J, Raj B. Effect of clustering on the thermal conductivity of nanofluids. Materials Chemistry and Physics 2008;109:50–5.
[17] Parekh K, Lee HS. Magnetic field induced enhancement in thermal conductivity of magnetite nanofluid. Journal of Applied Physics 2010;107:09 A310–13.
[18] Nasiri A, Shariaty-Niasar M, Rashidi A, Amrollahi A, Khodafarin R. Effect of dispersion method on thermal conductivity and stability of nanofluid. Experimental Thermal and Fluid Science 2011;35:717–23.
[19] Sundar LS, Sharma KV. Thermal conductivity enhancement of nanoparticles in distilled water. International Journal of Nanoparticles 2008;1:66–77.
[20] Mintsa HA, Roy G, Nguyen CT, Doucet D. New temperature dependent thermal conductivity data for water-based nanofluids. International Journal of Thermal Sciences 2009;48:363–71.
[21] Pang C, Jung JY, Lee JW, Kang YT. Thermal conductivity measurement of methanol-based nanofluids with Al_2O_3 and SiO_2 nanoparticles. International Journal of Heat and Mass Transfer 2012;55:5597–602.
[22] Saleh R, Putra N, Prakoso SP, Septiadi WN. Experimental investigation of thermal conductivity and heat pipe thermal performance of ZnO nanofluids. International Journal of Thermal Sciences 2013;63:125–32.
[23] Sajadi AR, Kazemi MH. Investigation of turbulent convective heat transfer and pressure drop of TiO_2/water nanofluid in circular tube. International Communications in Heat and Mass Transfer 2011;38:1474–8.
[24] Yang Y, Zhang ZG, Grulke EA, Anderson WB, Wu G. Heat transfer properties of nanoparticle in fluid dispersions (nanofluids) in laminar flow. International Journal of Heat and Mass Transfer 2005;48:1107–16.
[25] Xuan Y, Li Q. Heat transfer enhancement of nanofuids. International Journal of Heat and Fluid Flow 2000;21:58–64.
[26] Maiga SEB, Palm SJ, Nguyen CT, Roy G, Galanis N. Heat transfer enhancement by using nanofluids in forced convection flows. International Journal of Heat and Fluid Flow 2005;26:530–46.
[27] Arani AAA, Amani J. Experimental study on the effect of TiO_2–water nanofluid on heat transfer and pressure drop. Experimental Thermal and Fluid Science 2012;42:107–15.
[28] Mosavian MTH, Heris SZ, Etemad SGh, Esfahany MN. Heat transfer enhancement by application of nano-powder. Journal of Nanoparticle Research 2010;12:2611–9.
[29] Heris SZ, Esfahany MN, Etemad SGh. Experimental investigation of convective heat transfer of Al_2O_3/water nanofluid in circular tube. International Journal of Heat and Fluid Flow 2007;28:203–10.
[30] Sundar LS, Naik MT, Sharma KV, Singh MK, Reddy TCS. Experimental investigation of forced convection heat transfer and friction factor in a tube with Fe_3O_4 magnetic nanofluid. Experimental Thermal and Fluid Science 2012;37:65–71.
[31] Hwang KS, Jang SP, Choi SUS. Flow and convective heat transfer characteristics of water-based Al_2O_3 nanofluids in fully developed laminar flow regime. International Journal of Heat and Mass Transfer 2009;52:193–9.
[32] Ding Y, Alias H, Wen D, Williams RA. Heat transfer of aqueous suspensions of carbon nanotubes (CNT nanofluids). International Journal of Heat and Mass Transfer 2006;49:240–50.
[33] Sundar LS, Sharma KV. Turbulent heat transfer and friction factor of Al_2O_3 nanofluid in circular tube with twisted tape inserts. International Journal of Heat and Mass Transfer 2010;53:1409–16.
[34] Sundar LS, Sharma KV. Heat transfer enhancements of low volume concentration Al_2O_3 nanofluid and with longitudinal strip inserts in a circular tube. International Journal of Heat and Mass Transfer 2010;53:4280–6.
[35] Sundar LS, Kumar NTR, Naik MT, Sharma KV. Effect of full length twisted tape inserts on heat transfer and friction factor enhancement with Fe_3O_4 magnetic nanofluid inside a plain tube: an experimental study. International Journal of Heat and Mass Transfer 2012;55:2761–8.
[36] Suresh S, Chandrasekar M, Sekhar SC. Experimental studies on heat transfer and friction factor characteristics of CuO/water nanofluid under turbulent flow in a helically dimpled tube. Experimental Thermal and Fluid Science 2011;35:542–9.
[37] Suresh S, Selvakumar P, Chandrasekar M, Raman VS. Experimental studies on heat transfer and friction factor characteristics of Al_2O_3/water nanofluid under turbulent flow with spiraled rod inserts. Chemical Engineering and Processing 2012;53:24–30.
[38] Suresh S, Venkitaraj KP, Selvakumar P. Comparative study on thermal performance of helical screw tape inserts in laminar flow using Al_2O_3/water and CuO/water nanofluids. Superlattices and Microstructures 2011;49:608–22.
[39] Pathipakka G, Sivashanmugam P. Heat transfer behaviour of nanofluids in a uniformly heated circular tube fitted with helical inserts in laminar flow. Superlattices and Microstructures 2010;47:349–60.
[40] Saeedinia M, Behabadi MAA, Nasr M. Experimental study on heat transfer and pressure drop of nanofluid flow in a horizontal coiled wire inserted tube under constant heat flux. Experimental Thermal and Fluid Science 2012;36:158–68.
[41] Wongcharee K, Eiamsa-ard S. Enhancement of heat transfer using CuO/water nanofluid and twisted tape with alternate axis. International Communications in Heat and Mass Transfer 2011;38:742–8.
[42] Tzou DY. Thermal instability of nanofluids in natural convection. International Journal of Heat and Mass Transfer 2008;51:2967–79.
[43] Abu-Nada E, Masoud Z, Hijazi A. Natural convection heat transfer enhancement in horizontal concentric annuli using nanofluids. International Communications in Heat Mass Transfer 2008;35:657–65.
[44] Tahery AA, Pesteei SM, Zehforoosh A. Numerical study of heat transfer performance of homogenous nanofluids under natural convection. International Journal of Chemical Engineering and Applications 2010;1:49–54.
[45] Moghari RM, Akbarinia A, Shariat M, Talebi F, Laur R. Two phase mixed convection Al_2O_3–water nanofluid flow in an annulus. International Journal of Multiphase Flow 2011;37:585–95.
[46] Heris SZ. Experimental investigation of pool boiling characteristics of low-concentrated CuO/ethylene glycol–water nanofluids. International Communications in Heat and Mass Transfer 2011;38:1470–3.
[47] Henderson K, Park YG, Liu L, Jacobi AM. Flow-boiling heat transfer of R-134a-based nanofluids in a horizontal tube. International Journal of Heat and Mass Transfer 2010;53:944–51.
[48] Peng H, Ding G, Jiang W, Hu H, Gao Y. Heat transfer characteristics of refrigerant-based nanofluid flow boiling inside a horizontal smooth tube. International Journal of Refrigeration 2009;32:1259–70.
[49] Peng H, Ding G, Hu H, Jiang W, Zhuang D, Wang K. Nucleate pool boiling heat transfer characteristics of refrigerant/oil mixture with diamond nanoparticles. International Journal of Refrigeration 2010;33:347–58.

[50] Peng H, Ding G, Hu H, Jiang W. Effect of nanoparticle size on nucleate pool boiling heat transfer of refrigerant/oil mixture with nanoparticles. International Journal of Heat and Mass Transfer 2011;54:1839–50.
[51] Huminic G, Huminic A. Heat transfer characteristics in double tube helical heat exchangers using nanofluids. International Journal of Heat and Mass Transfer 2011;54:4280–7.
[52] Duangthongsuk W, Wongwises S. Heat transfer enhancement and pressure drop characteristics of TiO_2–water nanofluid in a double-tube counter flow heat exchanger. International Journal of Heat Mass Transfer 2009;52:2059.
[53] Farajollahi B, Etemad SGh, Hojjat M. Heat transfer of nanofluids in a shell and tube heat exchanger. International Journal of Heat and Mass Transfer 2010;53:12–7.
[54] Demir H, Dalkilic AS, Kürekci NA, Duangthongsuk W, Wongwises S. Numerical investigation on the single phase forced convection heat transfer characteristics of TiO_2 nanofluids in a double-tube counter flow heat exchanger. International Communications in Heat and Mass Transfer 2011;38: 218–28.
[55] Yousefi T, Veisy F, Shojaeizadeh E, Zinadini S. An experimental investigation on the effect of MWCNT–H_2O nanofluid on the efficiency of flat-plate solar collectors. Experimental Thermal and Fluid Science 2012;39:207–12.
[56] Yousefi T, Shojaeizadeh E, Veysi F, Zinadini S. An experimental investigation on the effect of pH variation of MWCNT–H_2O nanofluid on the efficiency of a flat-plate solar collector. Solar Energy 2012;86:771–9.
[57] Lotfi R, Rashidi AM, Amrollahi A. Experimental study on the heat transfer enhancement of MWNT–water nanofluid in a shell and tube heat exchanger. International Communications in Heat and Mass Transfer 2012;39:108–11.
[58] Yousefi T, Veysi F, Shojaeizadeh E, Zinadini S. An experimental investigation on the effect of $Al_2O_3eH_2O$ nanofluid on the efficiency of flat-plate solar collectors. Renewable Energy 2012;39:293–8.
[59] Peyghambarzadeh SM, Hashemabadi SH, Jamnani MS, Hoseini SM. Improving the cooling performance of automobile radiator with Al_2O_3/water nanofluid. Applied Thermal Engineering 2011;31:1833–8.
[60] Savithiri S, Pattamatta A, Das SK. Scaling analysis for the investigation of slip mechanisms in nanofluids. Nanoscale Research Letters 2011;6:471–86.
[61] White S, Shih A, Pipe K. Investigation of the electrical conductivity of propylene glycol-based ZnO nanofluids. Nanoscale Research Letters 2011;6: 346–51.
[62] Ijam A, Saidur R. Nanofluid as a coolant for electronic devices (cooling of electronic devices). Applied Thermal Engineering 2012;32:76–82.
[63] Murshed SMS, Leong KC, Yang C. Thermophysical and electrokinetic properties of nanofluids—a critical review. Applied Thermal Engineering 2008;28: 2109–1125.
[64] Chandrasekar M, Suresh S, Senthilkumar T. Mechanisms proposed through experimental investigations on thermophysical properties and forced convective heat transfer characteristics of various nanofluids—a review. Renewable and Sustainable Energy Reviews 2012;16:3917–38.
[65] Paul G, Chopkar M, Manna I, Das PK. Techniques for measuring the thermal conductivity of nanofluids: a review. Renewable and Sustainable Energy Reviews 2010;14:1913–24.
[66] Ghadimi A, Saidur R, Metselaar HSC. A review of nanofluid stability properties and characterization in stationary conditions. International Journal of Heat and Mass Transfer 2011;54:4051–68.
[67] Mahbubul IM, Saidur R, Amalina MA. Latest developments on the viscosity of nanofluids. International Journal of Heat and Mass Transfer 2012;55:874–85.
[68] Pak BC, Cho YI. Hydrodynamic and heat transfer study of dispersed fluids with submicron metallic oxide particles. Experimental Heat Transfer 1998;11: 151–70.
[69] Masuda H, Ebata A, Teramae K, Hishinuma N. Alteration of thermal conductivity and viscosity of liquid by dispersing ultra-fine particles (dispersion of -Al_2O_3, SiO_2 and TiO_2 ultra-fine particles). Netsu Bussei (Japan) 1993;4:227–33.
[70] Bobbo S, Fedele L, Benetti A, Colla L, Fabrizio M, Pagura C, et al. Viscosity of water based SWCNH and TiO_2 nanofluids. Experimental Thermal and Fluid Science 2012;36:65–71.
[71] Lee JH, Hwang KS, Jang SP, Lee BH, Kim JH, Choi SUS, et al. Effective viscosities and thermal conductivities of aqueous nanofluids containing low volume concentrations of Al_2O_3 nanoparticles. International Journal of Heat and Mass Transfer 2008;51:2651–6.
[72] Wang X, Xu X, Choi SUS. Thermal conductivity of nanoparticle–fluid mixture. Journal of Thermophysics Heat Transfer 1999;13:474–80.
[73] Chadwick MD, Goodwin JW, Vincent B, Lawson EJ, Mills PDA. Rheological behaviour of titanium dioxide (uncoated anatase) in ethylene glycol. Colloids and Surfaces A: Physicochemical and Engineering Aspects 2002;196:235–45.
[74] Kwak K, Kim C. Viscosity and thermal conductivity of copper oxide nanofluid dispersed in ethylene glycol. Korea–Australia Rheology Journal 2005;17:35–40.
[75] Teipel U, Forter-Barth U. Rheology of nano-scale aluminum suspensions. Propellants Explosives Pyrotechnics 2001;26:268–72.
[76] Katiyar A, Singh AN, Shukla P, Nandi T. Rheological behavior of magnetic nanofluids containing spherical nanoparticles of Fe–Ni. Powder Technology 2012;224:86–9.
[77] Prasher R, Song D, Wang J, Phelan P. Measurements of nanofluid viscosity and its implications for thermal applications. Applied Physics Letters 2006;89: 133108–11.
[78] Chen H, Witharana S, Jina Y, Kim C, Ding Y. Predicting thermal conductivity of liquid suspensions of nanoparticles (nanofluids) based on rheology. Particuology 2009;7:151–7.
[79] Murshed SMS, Leong KC, Yang C. Investigations of thermal conductivity and viscosity of nanofluids. International Journal of Thermal Sciences 2008;47: 560–8.
[80] Kulkarni DP, Das DK, Vajjha RS. Application of nanofluids in heating buildings and reducing pollution. Applied Energy 2009;86:2566–73.
[81] Naik MT, Sundar LS. Investigation into thermophysical properties of glycol based CuO nanofluid for heat transfer applications. World Academy of Science Engineering and Technology 2011;59:440–6.
[82] Sundar LS, M.dH. Farooky, Sarada SN, Singh MK. Experimental thermal conductivity of ethylene glycol and water mixture based low volume concentration of Al_2O_3 and CuO nanofluids. International Communications in Heat and Mass Transfer 2012;41:41–6.
[83] Das SK, Choi SUS, Patel H. Heat transfer in nanofluids—a review. Heat Transfer Engineering 2006;27:3–19.
[84] Keblinski P, Eastman J, Cahill D. Nanofluids for thermal transport. Materials Today 2005;8:36–44.
[85] Daungthongsuk W, Wongwises S. A critical review of convective heat transfer of nanofluids. Renewable and Sustainable Energy Reviews 2007;11:797–817.
[86] Sridhara V, Satapathy LN. Al_2O_3-based nanofluids: a review. Nanoscale Research Letter 2011;6:456.
[87] Wagener M, Murty BS, Gunther B. Preparation of metal nanosuspensions by high-pressure DC-sputtering on running liquids. In: Komarnenl S, Parker JC, Wollenberger HJ, editors. Nanocrystalline and nanocomposite materials II, 457. Pittsburgh: Materials Research Society; 1997. p. 149–54.
[88] Zhu H, Lin Y, Yin Y. A novel one-step chemical method for preparation of copper nanofluids. Journal of Colloid and Interface Science 2004;227:100–3.
[89] Zhu H, Han D, Meng Z, Wu D, Zhang C. Preparation and thermal conductivity of CuO nanofluid via a wet chemical method. Nanoscale Research Letters 2011;6:181–7.
[90] Lo CH, Tsung TT, Chen LC. Shape-controlled synthesis of Cubased nanofluid using submerged arc nanoparticle synthesis system (SANSS). Journal of Crystal Growth 2005;277:636–42.
[91] Lo CH, Tsung TT, Chen LC. Ni nano-magnetic fluid prepared by submerged arc nano synthesis system (SANSS). JSME International Journal, Series B: Fluids and Thermal Engineering 2006;48:750–5.
[92] Xie H, Wang J, Xi T, Liu Y, Ai F, Wu Q. Thermal conductivity enhancement of suspensions containing nanosized alumina particles. Journal of Applied Physics 2002;91:4568–72.
[93] Hong TK, Yang HS, Choi CJ. Study of the enhanced thermal conductivity of Fe nanofluids. Journal of Applied Physics 2005;97:1–4.
[94] Einstein A. Eine neue Bestimmung der Moleküldimensionen. Annalen der Physik 1906;19:289–306.
[95] Brinkman HC. The viscosity of concentrated suspensions and solution. Journal of Chemical Physics 1952;20:571–81.
[96] Frankel NA, Acrivos A. On the viscosity of a concentrate suspension of solid spheres. Chemical Engineering Science 1967;22:847–53.
[97] Lundgren TS. Slow flow through stationary random beds and suspensions of spheres. Journal of Fluid Mechanics 1972;51:273–99.
[98] Batchelor GK. The effect of Brownian motion on the bulk stress in a suspension of spherical particles. Journal of Fluid Mechanics 1977;83:97–117.
[99] Graham AL. On the viscosity of suspensions of solid spheres. Applied Science Research 1981;37:275–86.
[100] Krieger IM, Dougherty TJ. A mechanism for non-Newtonian flow in suspensions of rigid spheres. Transactions of the Society of Rheology 1959;3: 137–52.
[101] Chen H, Ding Y, He Y, Tan C. Rheological behaviour of ethylene glycol based titania nanofluids. Chemical Physics Letters 2007;444:333–7.
[102] Kitano T, Kataoka T, Shirota T. An empirical equation of the relative viscosity of polymer melts filled with various inorganic fillers. Rheological Acta 1981;20: 207–9.
[103] Nielsen LE. Generalized equation for the elastic moduli of composite materials. Journal of Applied Physics 1970;41:4626–7.
[104] Fullman RL. Measurement of particle sizes in opaque bodies. Journal of Metals 1953;5:447–52.
[105] Neto JBR. Deffloculation mechanisms of colloidal clay suspensions.Materials Science and Engineering (in Portuguese). Florianopolis: UFSC; 1999 Ph.D. thesis.
[106] A.D. Jr. Noni, Garcia DE, Hotza D. A modified model for the viscosity of ceramic suspensions. Ceramic International 2002;28:731–5.
[107] Chandrasekar M, Suresh S, Bose AC. Experimental investigations and theoretical determination of thermal conductivity and viscosity of Al_2O_3/water nanofluid. Experimental Thermal and Fluid Science 2010;34:210–6.
[108] Mooney M. Secondary stresses in viscoelastic flow. Journal of Colloid Science 1951;6:96–107.
[109] Thomas CU, Muthukumar M. Convergence of multiple scattering series for two-body hydrodynamic effects on shear viscosity of suspensions of spheres. Journal of Chemical Physics 1991;94:4557–68.
[110] Metzner AB. Rheology of suspensions in polymeric liquids. Journal of Rheology 1985;29:739–74.
[111] Leighton D, Acrivos A. The shear-induced migration of particles in concentrated suspensions. Journal of Fluid Mechanics 1987;181:415–39.
[112] Barnes HA, Hutton JF, Walters K. An introduction to rheology. Amsterdam, The Netherlands: Elsevier; 1989.
[113] Cheng NS, Law AWK. Exponential formula for computing effective viscosity. Powder Technology 2003;129:156–60.

[114] Graf WH. Hydraulics of sediment transport. New York: McGraw-Hill; 1971.
[115] Avsec J, Oblak M. The calculation of thermal conductivity, viscosity and thermodynamic properties for nanofluids on the basis of statistical nanomechanics. International Journal of Heat and Mass Transfer 2007;50: 4331–41.
[116] Yu W, Choi SUS. The role of interfacial layers in the enhanced thermal conductivity of nanofluids: a renovated Hamilton–Crosser model. Journal of Nanoparticle Research 2004;6:355–61.
[117] Tseng WJ, Chen CN. Effect of polymeric dispersant on rheological behaviour of nickel–terpineol suspensions. Material Science and Engineering A 2003;347: 145–53.
[118] Tseng WJ, Lin KC. Rheology and colloidal structure of aqueous TiO_2 nanoparticle suspensions. Material Science and Engineering A 2003;355:186–92.
[119] Rea U, McKrell T, Hu LW, Buongiorno J. Laminar convective heat transfer and viscous pressure loss of alumina–water and zirconia–water nanofluids. International Journal of Heat and Mass Transfer 2009;52:2042–8.
[120] Williams WC, Buongiorno J, Hu LW. Experimental investigation of turbulent convective heat transfer and pressure loss of alumina/water and zirconia/water nanoparticle colloids (nanofluids) in horizontal tubes. Journal of Heat Transfer 2008;130:042412.
[121] Nguyen C, Desgranges F, Roy G, Galanis N, Mare T, Boucher S, et al. Temperature and particle-size dependent viscosity data for water-based nanofluids—hysteresis phenomenon. International Journal of Heat Fluid Flow 2007;28:1492–506.
[122] Maiga SEB, Nguyen CT, Galanis N, Roy G. Heat transfer behaviours of nanofluids in a uniformly heated tube. Superlattices and Microstructures 2004;35:543–57.
[123] Godson L, Raja B, Lal DM, Wongwises S. Experimental investigation on the thermal conductivity and viscosity of silver-deionized water nanofluid. Experimental Heat Transfer 2010;23:317–32.
[124] Hagen KD. Heat transfer with applications. Prentice-Hall; 1999.
[125] White FM. Viscous fluid flow. New York: McGraw-Hill; 1991.
[126] Reid R, Prausnitz J, Sherwood T. The properties of gases and liquids. 4th ed. New York: McGraw-Hill; 1987.
[127] Yaws CL. Physical properties: a guide to the physical, thermodynamic and transport property data of industrially important chemical compounds. McGraw-Hill; 1977.
[128] Abu-Nada E. Effects of variable viscosity and thermal conductivity of Al_2O_3–water nanofluid on heat transfer enhancement in natural convection. International Journal of Heat Fluid Flow 2009;30:679–90.
[129] Hosseini SM, Moghadassi AR, Henneke DE. A new dimensionless group model for determining the viscosity of nanofluids. Journal of Thermal Analysis Calorimetry 2010;100:873–7.
[130] Masoumi N, Sohrabi N, Behzadmehr A. A new model for calculating the effective viscosity of nanofluids. Journal of Physics D 1999;42:055501.
[131] Duangthongsuk W, Wongwises S. Measurement of temperature dependent thermal conductivity and viscosity of TiO_2–water nanofluids. Experimental Thermal and Fluid Science 2009;33:706–14.
[132] Corcione M. Empirical correlating equations for predicting the effective thermal conductivity and dynamic viscosity of nanofluids. Energy Conversion and Management 2011;52:789–93.
[133] Arani AAA, Amani J. Experimental investigation of diameter effect on heat transfer performance and pressure drop of TiO_2–water nanofluid. Experimental Thermal and Fluid Science 2013;44:520–33.
[134] Vakili-Nezhaad G, Dorany A. Effect of single-walled carbon nanotube on the viscosity of lubricants. Energy Procedia 2012;14:512–7.
[135] Brenner H, Condiff DW. Transport mechanics in systems of orientable particles. Part IV. Convective transport. Journal of Colloid Interface Science 1974;47:199–264.
[136] Yu W, Xie H, Li Y, Chen L. Experimental investigation on thermal conductivity and viscosity of aluminum nitride nanofluid. Particuology 2011;9:187–91.
[137] Kulkarni DP, Das DK, Chukwu GA. Temperature dependent rheological property of copper oxide nanoparticles suspension (nanofluid). Journal of Nanoscience and Nanotechnology 2006;6:1150–4.
[138] Namburu PK, Kulkarni DP, Misra D, Das DK. Viscosity of copper oxide nanoparticles dispersed in ethylene glycol and water mixture. Experimental Thermal and Fluid Science 2007;32:67–71.
[139] Namburu PK, Das DK, Tanguturi KM, Vajjha RS. Numerical study of turbulent flow and heat transfer characteristics of nanofluids considering variable properties. International Journal of Thermal Sciences 2009;48:290–302.
[140] Namburu PK, Kulkarni DP, Dandekar A, Das DK. Experimental investigation of viscosity and specific heat of silicon dioxide nanofluids. Micro and Nano Letters 2007;2:67–71.
[141] Sahoo BC, Vajjha RS, Ganguli R, Chukwu GA, Das DK. Determination of rheological behavior of aluminum oxide nanofluid and development of new viscosity correlations. Petroleum Science and Technology 2009;27:1757–70.
[142] Vajjha RS, Das DK. A review and analysis on influence of temperature and concentration of nanofluids on thermophysical properties, heat transfer and pumping power. International Journal of Heat and Mass Transfer 2012;55: 4063–78.
[143] Kole M, Dey TK. Viscosity of alumina nanoparticles dispersed in car engine coolant. Experimental Thermal and Fluid Science 2010;34:677–83.
[144] Andrade END. A theory of the viscosity of liquids—Part I. Philosophical Magazine 1934;17:497–511.
[145] Kole M, Dey TK. Effect of aggregation on the viscosity of copper oxide–gear oil nanofluids. International Journal of Thermal Sciences 2011;50:1741–7.
[146] Sundar LS, Ramana EV, Singh MK, Sousa ACM. Viscosity of low volume concentrations of magnetic Fe_3O_4 nanoparticles dispersed in ethylene glycol and water mixture. Chemical Physics Letters 2012;554:236–42.
[147] Duan F, Wong TF, Crivoi A. Dynamic viscosity measurement in non-Newtonian graphite nanofluids. Nanoscale Research Letters 2012;7:360–6.
[148] Putra N, Roetzel W, Das SK. Natural convection of nano-fluids. Heat Mass Transfer 2003;39:775–84.
[149] Schmidt AJ, Chiesa M, Torchinsky DH, Johnson JA, Boustani A, McKinley GH, et al. Experimental investigation of nanofluid shear and longitudinal viscosities. Applied Physics Letters 2008;92:244107.
[150] Phuoc TX, Massoudi M. Experimental observations of the effects of shear rates and particle concentration on the viscosity of Fe_2O_3-deionized water nanofluids. International Journal of Thermal Science 2009;48:1294–301.
[151] Anoop KB, Sundararajan T, Das SK. Effect of particle size on the convective heat transfer in nanofluid in the developing region. International Journal of Heat Mass Transfer 2009;52:2189–95.
[152] Lee SW, Park SD, Kang S, Bang IC, Kim JH. Investigation of viscosity and thermal conductivity of SiC nanofluids for heat transfer applications. International Journal of Heat Mass Transfer 2011;54:433–8.
[153] Zhu H, Li C, Wu D, Zhang C, Yin Y. Preparation characterization viscosity and thermal conductivity of $CaCO_3$ aqueous nanofluids. Science China Technology Sciences 2010;53:360–8.
[154] Duan F, Kwek D, Crivoi A. Viscosity affected by nanoparticle aggregation in Al_2O_3–water nanofluids. Nanoscale Research Letters 2011;6:248–53.
[155] Chen L, Xie H, Li Y, Yu W. Nanofluids containing carbon nanotubes treated by mechanochemical reaction. Thermochimica Acta 2008;477:21–4.
[156] Anoop KB, Kabelac S, Sundararajan T, Das SK. Rheological and flow characteristics of nanofluids: influence of electroviscous effects and particle agglomeration. Journal of Applied Physics 2009;106:034909.
[157] Pastoriza-Gallego MJ, Casanova C, Legido JL, Piñeiro MM. CuO in water nanofluid: influence of particle size and polydispersity on volumetric behaviour and viscosity. Fluid Phase Equilibria 2011;300:188–96.
[158] Turgut A, Tavman I, Chirtoc M, Schuchmann HP, Sauter C, Tavman S. Thermal conductivity and viscosity measurements of water-based TiO_2 nanofluids. International Journal of Thermophysics 2009;30:1213–26.
[159] Nguyen C, Desgranges F, Galanis N, Roy G, Mare T, Boucher S, et al. Viscosity data for Al_2O_3–water nanofluid—hysteresis: is heat transfer enhancement using nanofluids reliable? International Journal of Thermal Science 2008;47:103–11.
[160] Saeedinia M, Akhavan-Behabadi MA, Razi P. Thermal and rheological characteristics of CuO–base oil nanofluid flow inside a circular tube. International Communications in Heat and Mass Transfer 2012;39:152–9.
[161] Ferrouillat S, Bontemps A, Ribeiro JP, Gruss JA, Soriano O. Hydraulic and heat transfer study of SiO_2/water nanofluids in horizontal tubes with imposed wall temperature boundary conditions. International Journal of Heat and Fluid Flow 2011;32:424–39.
[162] Timofeeva EV, Moravek MR, Singh D. Improving the heat transfer efficiency of synthetic oil with silica nanoparticles. Journal of Colloid and Interface Science 2011;364:71–9.
[163] Yu W, Xie H, Chen L, Li Y. Investigation of thermal conductivity and viscosity of ethylene glycol based ZnO nanofluid. Thermochimica Acta 2009;491:92–6.
[164] Yu W, France DM, Smith DS, Singh D, Timofeeva EV, Routbort JL. Heat transfer to a silicon carbide/water nanofluid. International Journal of Heat and Mass Transfer 2009;52:3606–12.
[165] Buschmann MH. Thermal conductivity and heat transfer of ceramic nanofluids. International Journal of Thermal Sciences 2012;62:19–28.
[166] Aladag B, Halelfadl S, Doner N, Maré T, Duret S, Estellé P. Experimental investigations of the viscosity of nanofluids at low temperatures. Applied Energy 2012;97:876–80.
[167] Hojjat M, Etemad SGh. Bagheri R, Thibault J. Convective heat transfer of non-Newtonian nanofluids through a uniformly heated circular tube. International Journal of Thermal Sciences 2011;50:525–31.
[168] He Y, Jin Y, Chen H, Ding Y, Cang D, Lu H. Heat transfer and flow behavior of aqueous suspensions of TiO_2 nanoparticles (nanofluids) flowing upward through a vertical pipe. International Journal of Heat and Mass Transfer 2007;50:2272–81.
[169] Chevalier J, Tillement O, Ayela F. Rheological properties of nanofluids flowing through microchannels. Applied Physics Letters 2007;91:233103.
[170] Timofeeva EV, Routbort JL, Singh D. Particle shape effects on thermophysical properties of alumina nanofluids. Journal of Applied Physics 2009;106: 014304.
[171] Tseng WJ, Wu CH. Aggregation, rheology and electrophoretic packing structure of aqueous $A1_2O_3$ nanoparticle suspensions. Acta Materialia 2002;50: 3757–66.
[172] Mosavian MTH, Heris SZ, Etemad SGh, Esfahany MN. Heat transfer enhancement by application of nano-powder. Journal of Nanoparticle Research 2010;12:2611–9.
[173] Kulkarni DP, Das DK, Vajjha RS. Application of nanofluids in heating buildings and reducing pollution. Applied Energy 2009;86:2566–73.
[174] Lu W, Fan Q. Study for the particle's scale effect on some thermophysical properties of nanofluids by a simplified molecular dynamics method. Engineering Analysis with Boundary Element 2008;32:282–9.
[175] Garg J, Poudel B, Chiesa M, Gordon J, Ma J, Wang J, et al. Enhanced thermal conductivity and viscosity of copper nanoparticles in ethylene glycol nanofluid. Journal of Applied Physics 2008;103:074301.
[176] Tsai TH, Kuo LS, Chen PH, Yang CT. Effect of viscosity of base fluid on thermal conductivity of nanofluids. Applied Physics Letters 2008;93:233121.

[177] Phuoc TX, Massoudi M, Chen RH. Viscosity and thermal conductivity of nanofluids containing multi-walled carbon nanotubes stabilized by chitosan. International Journal of Thermal Science 2011;50:12–8.
[178] Chen H, Yang W, He Y, Ding Y, Zhang L, Tan C, et al. Heat transfer and flow behavior of aqueous suspensions of titanate nanotubes (nanofluids). Powder Technology 2008;183:63–72.
[179] Chen H, Ding Y, Lapkin A, Fan X. Rheological behaviour of ethylene glycol–titanate nanotube nanofluids. Journal of Nanoparticle Research 2009;11: 1513–20.
[180] Moosavi M, Goharshadi EK, Youssefi A. Fabrication, characterization, and measurement of some physicochemical properties of ZnO nanofluids. International Journal of Heat and Fluid Flow 2010;31:599–605.
[181] Amrollahi A, Rashidi AM, Lotfi R, Meibodi ME, Kashefi K. Convection heat transfer of functionalized MWNT in aqueous fluids in laminar and turbulent flow at the entrance region. International Communications in Heat and Mass Transfer 2010;37:717–23.
[182] Venerus DC, Buongiorno J, Christianson R, Townsend J, Bang IC, Chen G, et al. Viscosity measurements on colloidal dispersions (Nanofluids) for heat transfer applications. Applied Rheology 2010;20:44582.
[183] Tseng WJ, Lin CL. Effect of dispersants on rheological behavior of $BaTiO_3$ powders in ethanol–isopropanol mixtures. Material Chemistry Physics 2003;80: 232–8.